*Article*
# Metric Dimension of Maple Leaf Graph

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**Abstract.** The Maple Leaf Graph ($M_p$) is a newly introduced recursive graph constructed from a base cycle $C_6$ through a stepwise expansion process, where $p \geq 2$. The construction begins with a single cycle $C_6$, followed by the addition of one vertex to extend the structure. Subsequently, three vertices are added to form two $C_6$ graphs, and another three vertices are added to generate a third $C_6$. This recursive process continues until the Maple Leaf Graph ($M_p$) is fully formed. This study determines the metric dimension of the Maple Leaf Graph ($M_p$) for $2 \leq p \leq 9$ using the concepts of vertex distance and resolving sets. By analyzing the distance representation of each vertex with respect to a resolving set, the minimum resolving set is identified, defining the metric dimension of the graph. Calculations were performed manually to ensure consistency and accuracy. The analysis reveals a tiered linear reduction pattern, where the metric dimension does not increase linearly with p. The main findings are summarized in three theorems: for $p = 2$ and $p = 3$, the metric dimension of the Maple Leaf Graph equals p; for $p = 4, 5$, and $6$, it equals $p - 1$; and for $p = 7, 8$, and $9$, it equals $p - 2$. These results introduce a new class of graphs and provide theoretical insights into the behavior of metric dimension in multi-cycle constructions, thereby contributing to the development of combinatorial graph theory.

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## 1. Introduction

Combinatorics is a branch of mathematics that studies finite set structures and has a broad and fundamental scope. One of the important subfields of combinatorics is graph theory, first introduced by Euler through the Königsberg Bridge problem [1]. Subsequent developments were made by Kirchhoff in 1847 through his study of electrical networks, which gave rise to the theory of trees [2]. One of the key concepts in graph theory is the metric dimension, through the definition of resolving sets and vertex distances [3]. This concept has practical applications in robot navigation, location determination in networks, communication systems, and chemical structure representation [4].

Recent studies have investigated the metric dimension of various classes of graphs. There are amalgamation of triangular ladder graphs [5], spider web graphs constructed from star and cycle graphs [6], identified graphs [7], the Buckminsterfullerene graph[8], and the windmill graph [9]. Other works include studies on the metric dimension of corona product graphs [10], circulant graphs [11], cubic carbon crystal structures [12], as well as variants such as fault-tolerant metric dimension [13], dominant metric dimension [14-15], and local metric dimension [16]. More recent research has also connected the metric dimension with graph products, subdivisions, and complex chemical graphs [17,18]. However, most of these studies remain focused on well-known graphs or modifications of classical graphs, leaving the exploration of newly constructed graphs relatively limited.

In the past five years, further developments have enriched the study of metric dimension. For instance, recent work has computed the edge metric dimension of various networks with applications to chemistry [19], while fault-tolerant metric dimension has been analyzed in material structures such as GeSbTe superlattices [20]. Other studies determined the dominant metric dimension of wheel- and gear-based graphs [21], as well as the local metric dimension for graphs with small clique numbers [22]. Investigations have also addressed the metric dimension of line graphs of Bakelite and subdivided Bakelite networks [23]. Computational approaches have emerged, such as machine learning techniques to compute the metric dimension of graphs [24], and fractal cubic networks where metric and fault-tolerant variants were derived [25]. Moreover, dominant metric dimension has also been explored in zero-divisor graphs [26], while applications to chemical structures have been highlighted through vertex-based resolvability parameters for molecular identification [27]. Finally, fault-tolerant metric dimension in carbon networks has been investigated to assess resilience in chemical graph models [28]. These diverse directions emphasize the vitality of metric dimension research and its relevance to both theoretical and applied domains[29-30].

This study introduces the Maple Leaf Graph (Mp), a new graph constructed through a stepwise expansion of the cycle graph $C_6$ by adding vertices to form a leaf-like structure. Since this graph has not been previously studied, analyzing its metric dimension provides a novel contribution to the development of combinatorial studies. The aim of this research is to determine the metric dimension of the Maple Leaf Graph (Mp) for $2 \leq p \leq 9$, to formulate general patterns in the form of theorems, and to highlight the novelty of this graph compared to previous studies.

Suppose there is a maple leaf graph with the following vertex and edge sets.

$$V(M_p) = \{v_0, v_p, v_j, a_1, a_2, b_1, b_2, w_j, w_{j1}, w_{j2}, w_{j3}, w_{j4}, w_{j5}\} \tag{1}$$

$$E(M_p) = \{v_0 b_1, v_{b_2}, v_0 w_{j3}, v_1 a_1, v_p a_2, a_i b_1, a_2 b_2, w_j w_{wj1}, w_j w_{j5}, \tag{2}$$

$$w_{j1} w_{j2}, w_{j4} w_{j5}, w_{j3} w_{j2}, w_{j3} w_{j4}, v_j w_{j2}, v_{j+1} w_{j4} | 1 \leq j \leq p-1\}, \text{ see Figure 1.}$$
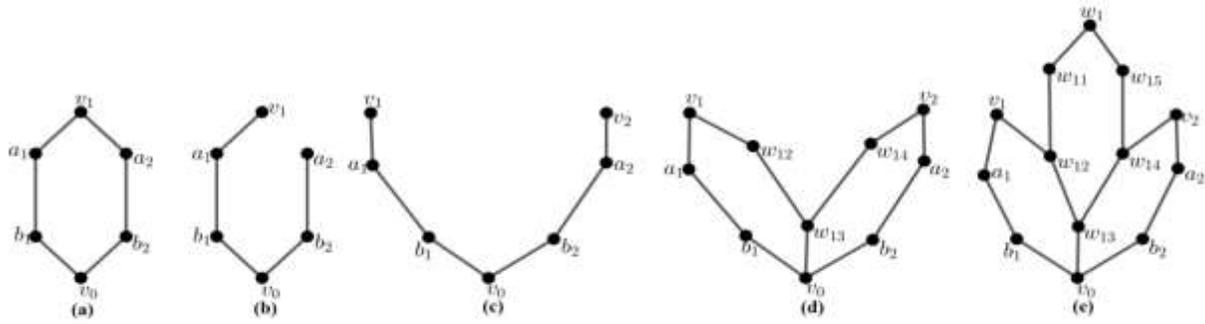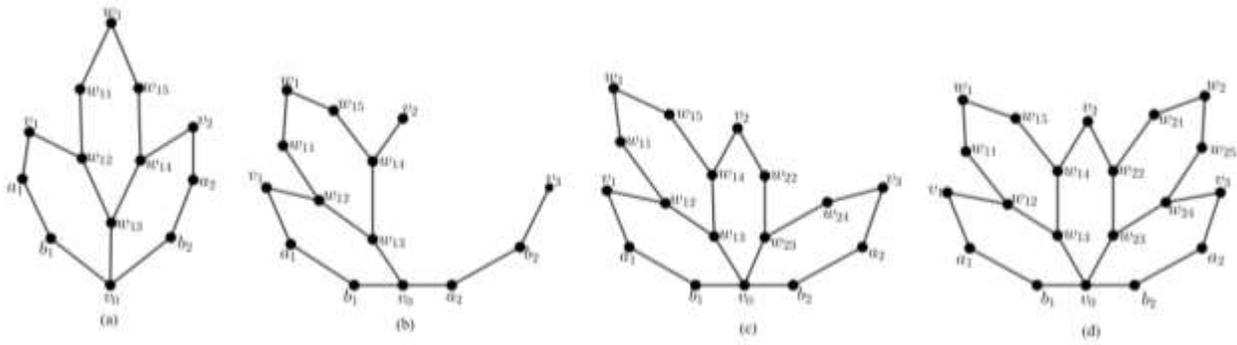
**Figure 1**. The Formation of the Maple Leaf Graph (M₂)



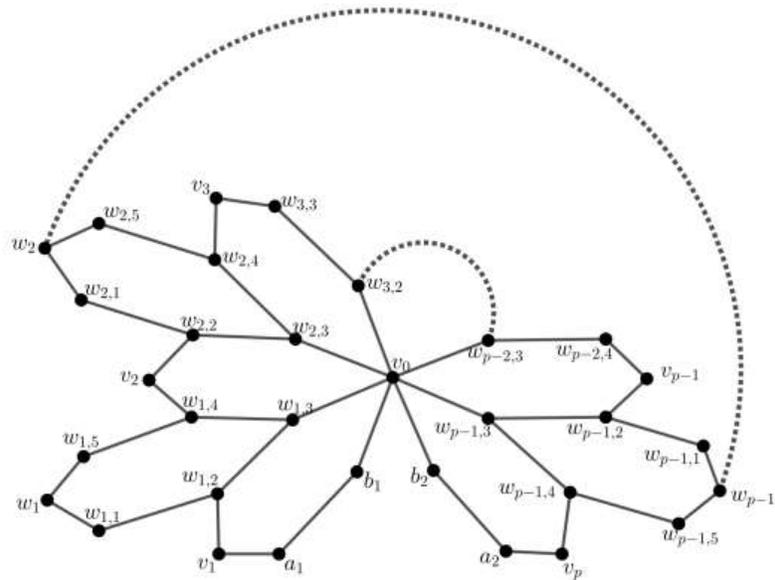**Figure 2.** The Formation of the Maple Leaf Graph (M₃)



**Figure 3.** The Generalization of the Maple Leaf Graph ($M_p$)

## 2. Research Method

Concept of metric dimension is a partition set vertices problem in graph. Before we determine metric dimension some graph, we must construct that graph completely. We define vertices and edges set of graph. For its metric dimension, we construct some subset of vertices until get as resolving set. Number of resolving set is upper bound of metric dimension that graph. For lower bound, if number Number of resolving set is k, we adjust to k-1 the definitions of metric dimension is not fulfilled [3-4].

### 2.1. Graph Construction

The Maple Leaf Graph $M_p$ is constructed recursively from the cycle graph $C_6$ as follows for p = 2, add one vertex to form an extended $C_6$. For p = 3, add three new vertices to connect two $C_6$ cycles. For p ≥ 4, add three vertices at each step to create an additional $C_6$ cycle. This process continues until the Maple Leaf Graph $M_p$ is obtained.

### 2.2. Determination of Metric Dimension

The determination follows the *upper–lower bound* approach Upper Bound: Construct a resolving set W with $|W|$ = k such that every vertex has a unique representation r(v|W). Lower bound: Show that any resolving set with $|W|$ = k−1 causes at least two vertices to share identical representations. When both conditions are satisfied, dim $(M_p)$ = k.

### 2.3 Verification of Results

Manual calculations of vertex distances were cross-verified using **Python NetworkX**, ensuring the computed distance matrices matched theoretical results. The code verified pairwise distances and resolving set uniqueness for each p ($2 \leq p \leq 9$).

### 2.4 Methodological Flow

The research procedure followed systematic stages as shown below:

#### 2.4.1. Stage 1 — Preliminary Phase

Literature review on metric dimension theory and recursive graph families. Definition and formulation of the Maple Leaf Graph ($M_p$). Development of initial conjecture on its metric dimension.

#### 2.4.2. Stage 2 — Implementation Phase

Stepwise construction of $M_p$ for $2 \leq p \leq 9$. Determination of distance matrices and resolving sets. Calculation of metric dimension using upper–lower bound verification.

#### 2.4.3. Stage 3 — Verification and Evaluation Phase

Validation of results was carried out manually to ensure the accuracy and consistency of distance calculations. Analysis of metric dimension patterns. Formulation of theorems and conjectures based on findings. In Figure 4 explain about flowchat determine metric dimension of graph mapple.

**Figure 4.** Flow Chat Metric Dimension of Graph Mapple

## 3.  Results and Discussion

The metric dimension as follows suppose there is a connected graph G = (E, V). Let there be an ordered set W = {$w_1, w_2, \dots, w_k$} ⊆ V(G). The representation of a vertex v ∈ V(G) with respect to W is denoted by r(v|W) = (d(v,$w_1$), d(v,$w_2$), …, d(v, $w_k$)). If every two vertices u and v satisfy r(u|W) ≠ r(v|W), the W is defined as a resolving set for G. A minimum resolving set is a resolving set with minimum cardinalty. The number of elements in the minimum resolving set id the metric dimension, denoted by dim(G) [31].

There are some basic results of metric dimension [2],[4],[17],[31]. For any connected graph G of order n ≥ 2, 1 ≤ dim(G) ≤n− 1. Moreover, dim($P_n$) = 1 if and only if G is a path graph, and dim($K_n$) = n − 1 if and only if G is a complete graph. Metric dimension of some graphs; for cycle graph, Cn, (n ≥ 3), we have dim($C_n$) = 2. For complete graph,  $K_n$, we have dim($K_n$) = n − 1. For star graph, $S_n$, we have dim($S_n$) = n − 2.

This study establishes the metric dimension pattern of the Maple Leaf graph $M_p$ for 2 ≤ p ≤ 9. By constructing the minimum resolving set, three main patterns are obtained:

$$dim\,(M_P) = \begin{cases} p, & 2 \le p \le 3, \\ p-1, & 4 \le p \le 6, \\ p-2, & 7 \le p \le 9. \end{cases}$$

**Table 1.** Summary of the metric dimension of the Maple Leaf Graph

| p | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| dim($M_p$) | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 7 |

To provide a clearer illustration of the observed pattern, Figure 5 depicts the placement of the resolving set in the maple leaf graphs $M_2$ and $M_3$
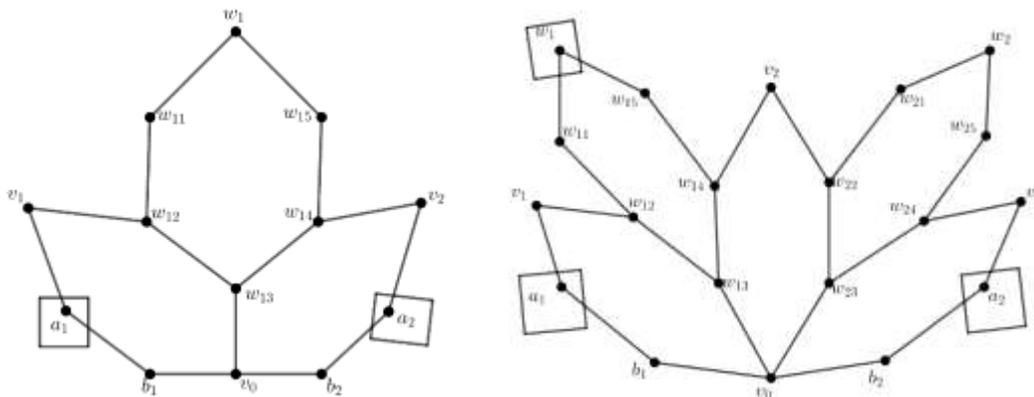


**Figure 5.** The resolving set in the maple leaf graphs $M_2$ and $M_3$

Based on Figure 5, an example of the vertex representation of the graphs $M_2$ and $M_3$ is provided, along with a proof of their metric dimension.

**Table 2.** Representation of vertices of $M_2$

| v | $r(v|w)$ | v | $r(v|w)$ | v | $r(v|w)$ | v | $r(v|w)$ |
|---|---|---|---|---|---|---|---|
| $a_1$ | (0, 4) | $a_2$ | (4, 0) | $b_1$ | (1, 3) | $b_2$ | (3, 1) |
| $v_0$ | (2, 2) | $v_1$ | (1, 5) | $v_2$ | (5, 1) | $w_1$ | (4, 4) |
| $w_{11}$ | (3, 5) | $w_{12}$ | (2, 4) | $w_{13}$ | (3, 3) | $w_{14}$ | (4, 2) |
| $w_{\{15]}$ | (5, 3) | | | | | | |

From Table 2, it can be seen that the representation of each vertex in $M_2$ is different. Consequently, we obtain $dim(M_2) \le 2$. Since $M_2$ is not a path graph, we have $dim(M_2) \ge 2$. Therefore $dim(M_2) = 2$

**Table 3.** Representation of vertices of $M_3$

| v | $r(v|w)$ | v | $r(v|w)$ | v | $r(v|w)$ | v | $r(v|w)$ |
|---|---|---|---|---|---|---|---|
| $a_1$ | (0, 4, 4) | $a_2$ | (4, 0, 6) | $b_1$ | (1, 3, 5) | $b_2$ | (3, 1, 5) |
| $v_0$ | (2, 2, 4) | $v_1$ | (1, 5, 3) | $v_2$ | (5, 5, 3 ) | $v_3$ | (5, 1, 7) |
| $w_1$ | (4, 6, 0) | $w_2$ | (6, 4, 6) | $w_{11}$ | (3, 5, 1) | $w_{12}$ | (2, 4, 2) |
| $w_{13}$ | (3, 3, 3) | $w_{14}$ | (4, 4, 2) | $w_{15}$ | (5, 5, 1) | $w_{21}$ | (5, 5, 5) |
| $w_{22}$ | (4, 4, 4) | $w_{23}$ | (3, 3, 5) | $w_{24}$ | (4, 2, 6) | $w_{25}$ | (5, 3, 7) |

From Table 3, it can be seen that the representation of all vertices in $M_3$ is different, thus we obtain $dim(M_3) \le 3$. Without loss of generality, if the cardinality of W is reduced by 1, then there will always

be two vertices having the same representation, namely the vertices that have distance 1 to $w_0$ Hence, $\dim(M_3) \geq 3$, therefore $\dim(M_{3)} = 3$.

These findings differend from previous studies. For example, in the Windmill Graph $K_1 + nK_3$ [9], the metric dimension increases linearly with the number of cycles at a constant gradient. In contrast, the Maple Leaf Graph exhibits a "slowdown" in growth starting from $p = 4$, shifting from the p pattern to $p - 1$ and then $p - 2$. This also differs from the Spider Web Graph [6], where the metric dimension is more influenced by radial connectivity rather than the number of added cycles.

The novelty of this research lies in the discovery of a tiered linear shift pattern in the metric dimension of the Maple Leaf Graph, which has not been observed in similar graph constructions (such as triangular ladder amalgamations, Buckminsterfullerene graphs, or tree identification graphs). This pattern shows that adding $C_6$ cycle blocks does not always increase the metric dimension fully, but instead follows a systematic reduction.

Thus, the contribution of this study is not only the determination of $\dim(M_p)$ for $2 \leq p \leq 9$, but also the identification of a new general pattern that opens opportunities for further studies on the relationship between multi-cycle structures and metric dimension behavior.

## 4. Conclusion

In this research, the metric dimension of the Maple Leaf Graph $M_p$ follows the pattern:

$$\dim(M_p)=p \text{ for } p=2,3; \quad \dim(M_p)=p-1 \text{ for } p=4,5,6; \quad \dim(M_p)=p-2 \text{ for } p=7,8,9.$$

These results reveal a tiered linear reduction pattern that differs from other previously studied graphs, such as Windmill graphs, Spider Web graphs, and amalgamation graphs, thereby providing a new theoretical contribution to the study of metric dimensions in cycle-based constructed graphs. The main contribution of this research is the introduction of a new graph, namely the Maple Leaf Graph, along with the discovery of its unique metric dimension characteristics, which do not entirely follow a simple linear pattern. This finding opens avenues for further research, including extending the calculation of the metric dimension of $M_p$ for $p > 9$, examining its relationship with other graph parameters such as diameter, domination number, or connectivity, and exploring its practical applications in network design with repetitive structures.

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