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Article

Quota Share Reinsurance and Excess of Loss Reinsurance Calculations Using Ruin's Theory

Dwi Susanti[1*], Riaman[1], Badrulfallah[1], Dimas Apriliyanto[1]

[1]Department of Mathematics, Faculty of Mathematics and Natural Science, Universitas Padjadjaran, Bandung, Indonesia

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Abstract. Ruin theory is commonly used to predict the likelihood of bankruptcy for an insurance company and relates to the rate of surplus of the insurance company for the insurance policy portfolio. Considering the change in the insurance fund from time to time, the timing of the occurrence of a number of claims is highly taken into account. Ruin theory is necessary so that companies can anticipate and detect bankruptcy early. One way to help insurance companies minimize their bankruptcy chances is through reinsurance. In this paper, will discuss about application of ruin theory in computing two methods of reinsurance treaty, that is Quota Share Reinsurance and Excess of Loss Reinsurance to decide more effective method to minimize probability of ruin. Results show that Excess of Loss Reinsurance method more effective than Quota Share Reinsurance method to minimize ruin probability of insurance company.

Corresponding Author :
Dwi Susanti
Department of Mathematics, Faculty of Mathematics and Natural Science,
Universitas Padjadjaran, Bandung, Indonesia
Email: dwi.susanti@unpad.ac.id

1. Introduction

The insurance industry is a financial sector that is highly susceptible to risk, particularly the risk associated with claims exceeding the premiums collected [1-2]. Insurance companies face significant challenges in ensuring that the premiums they collect are sufficient to cover the claims made by policyholders [3]. An imbalance between the amount of claims and premiums can lead to the bankruptcy of an insurance company, posing a serious threat to its financial stability [4]. Therefore, effective risk management approaches are crucial in this industry.

Ruin theory has emerged as an essential analytical tool for predicting the probability of an insurance company's bankruptcy. This theory provides a mathematical framework that allows insurance companies to measure the likelihood of ruin based on the distribution of claims and premiums received [5-6]. By utilizing models from ruin theory, insurance companies can conduct simulations and scenario analyses to assess their resilience to high claim fluctuations. Consequently, ruin theory forms a critical foundation in the risk management strategies of insurance companies [7-8].

To mitigate the risk of bankruptcy, many insurance companies rely on the services of reinsurance companies. Reinsurance is a mechanism through which insurance companies transfer part of the risks they face to reinsurance companies. In this way, if a large claim exceeds the financial capacity of the insurance company, the reinsurance company will cover a portion of that claim. This helps insurance companies maintain liquidity and financial stability. The role of reinsurance becomes increasingly important when the risk of claims faced by insurance companies rises, particularly in unstable economic conditions [9-11].

In reinsurance practice, there are two primary methods used: proportional and non-proportional reinsurance. Proportional reinsurance, such as Quota Share Reinsurance, allows for the proportional sharing of premiums and claims between the insurance company and the reinsurer. On the other hand, non-proportional reinsurance, such as Excess of Loss Reinsurance, involves the reinsurer covering losses that exceed a certain threshold. Selecting the appropriate method is crucial in ensuring that insurance companies can minimize their risk of ruin [12-14].

To determine the most effective reinsurance method for reducing the risk of bankruptcy, a thorough mathematical analysis using ruin theory is required. This study aims to evaluate and compare the effectiveness of various reinsurance methods in the context of reducing the probability of bankruptcy for insurance companies. The results of this research are expected to contribute to the development of better risk management strategies for insurance companies, enabling them to sustain their operations amid market uncertainties.

2. Problem Formulation

The ruin *theory* equation is defined as

$$\psi(u) = \frac{\exp(-Ru)}{E[\exp(-RU(T)|T < \infty)]} \tag{1}$$

where is the *adjudication coefficient*, which is the smallest positive number that satisfies the equation.

$$M_{S(t)-ct}(r) = 1 \tag{2}$$

If the Poisson distribution process is compounded with the average number of claims λt, then the equation will be obtained

$$\lambda[M_x(r) - 1] = cr \tag{3}$$

The claim equation for the *Quota Share Reinsurance* method is as follows:

$$h(x) = \alpha . x \qquad ; 0 \leq \alpha \leq 1 \tag{4}$$

and the claim equation for the *excess loss reinsurance* method is

$$\begin{aligned} h(x) &= 0 \quad ; x \leq \beta \\ &= x - \beta \quad ; x > \beta \end{aligned} \tag{5}$$

The sum of claims that must be paid by insurance companies that previously amounted to x, So after reinsurance is carried out, the number of claims to be paid will be as large as $x - h(x)$ and premiums received by insurance companies that were previously equal to c, After reinsurance is carried out, it will be as large as $c - C_h$. Therefore, in equation (3), the value of *its adjudication coefficient* R or hereinafter referred to as R_h . That is, the value of the *adjudication coefficient* after reinsurance can be written as a solution to the equation.

$$\lambda[M_{x-h(x)}(r) - 1] = (c - C_h)r \tag{6}$$

The problem to be discussed is to apply *ruin theory* in calculating the *Quota Share Reinsurance* and *Excess of Loss Reinsurance methods*, namely by finding the value of the *adjudication coefficient R* for each method, and then determine the method that is more effective in minimizing the chances of an *insurance company's ruin* by finding the maximum R value.

2.1 Claim Process

Suppose $N(t)$ state the number of claims and $S(t)$ the sum of aggregate claims paid up to time . The calculation begins at the time of $t = 0$ so that $N(0) = 0$. Next $S(t) = 0$ same as $N(t) = 0$. Suppose X_i state the size of the claim to-i, then:

$$S(t) = X_1 + X_2 + X_3 + \cdots + X_{N(t)} \tag{7}$$

Process $\{N(t), t \geq 0\}$ called the process of multiple claims, while $\{S(t), t \geq 0\}$ is the aggregate claims process (pool), e.g. $t \geq 0$ and $h > 0$, then $N(t+h) - N(t)$ is the multiplicity of claims and $S(t+h) - S(t)$ is the number of aggregate claims that occurred in a time interval t and $t + h$.

Next suppose T_i is the time when the claim to-i happen. Become $T_1, T_2, T_3 \ldots$ is a random variable and $T_1 < T_2 < T_3 < \cdots$. Therefore, it is unlikely that two or more claims will occur at the same time.

Function $N(t)$ and $S(t)$ is a function in the form of an ascending ladder where the discontinuity occurs at the time of T_i when a claim occurs and the ladder unit of measure for $N(t)$ excist 1 and $S(t)$ as big as X_i.

2.2 Surplus Function

The surplus function at the time t is defined as follows :

$$U(t) = u + c(t) - S(t) \qquad ; t \geq 0 \tag{8}$$

Suppose:

- u is the capital (initial surplus) at the time of $t = 0$
- $c(t)$ is a premium that is obtained continuously until time t
- $S(t)$ is an aggregate claim that has been paid up to time to- t

Surplus can be said to be the difference between the amount of premium received up to time to- t plus the initial capital is reduced by the sum of claims to be paid [15-16].

2.3 Ruin Theory

Ruin theory in the relationship between surplus processes $\{U(t); t \geq 0\}$ and $\{S(t); t \geq 0\}$ It has been explained in the previous discussion. Now assume that $S(t)$ is an aggregate claims process that is compound Poisson distributed [17-18]. From these assumptions, we can develop upper and lower limits on $\psi(u)$. In the special case of exponentially distributed individual claims, there is an explicit form of $\psi(u)$ [19-20].

The expected payment of claims per unit of time is $E[S(t)] = \lambda p_1$ where p_1 is a large average of claims. Assume the average premium received is greater than the expected claim payment per unit at that time, $c > \lambda p_1$. Next is the *relative security loading* θ is defined as follows:

$$c = (1 + \theta)\, \lambda p_1 \tag{9}$$

where $\theta > 0$, so if $\theta = 0$ or $\theta < 0$ cause $\psi(u) = 1$, Then it can be said that *ruin* must occur.

If $U(t)$ is a surplus process, $S(t)$ is a process of aggregate claims distributed by compound Poissons, with $c > \lambda p_1$, and $c = (1 + \theta)\, \lambda p_1$, then for $u \geq 0$ is a:

$$\psi(u) = \frac{\exp(-Ru)}{E[\exp(-RU(T)|T < \infty)]}$$

where R is the smallest positive number that satisfies the equation$R$

$$1 + (1 + \theta)p_1 r = M_x(r)$$

2.4 Maximum Aggregate Loss

The maximum aggregate loss random variable is defined as,

$$L = \max_{t \geq 0} \{S(t) - ct\} \tag{10}$$

is the maximum excess caused by aggregate claims far exceeding the amount of premium received [21-22], because $S(t) - ct = 0$, then for $t = 0$ Cause $L \geq 0$. To get a random variable function L, for $u \geq 0$, The relationship between maximal aggregate losses and *ruin* odds is as follows :

$$\begin{aligned}
1 - \psi(u) \quad &= \Pr[U(t) \geq 0, \forall\, t] \\
&= \Pr[u + ct - S(t) \geq 0, \forall\, t] \\
&= \Pr[S(t) - ct \leq u, \forall\, t] \\
&= \Pr\left[\max_{t \geq 0} \{S(t) - ct\} \leq u, \forall\, t\right] \\
&= \Pr[L \leq u]
\end{aligned}$$

So we have something in common,

$$1 - \psi(u) = \Pr\,[L \leq u], \text{ for } u \geq 0$$

for $t = 0$, then $S(t) - ct = S(0) - c.0 = 0$ so that $L \geq 0$. If $u = 0$, $1 - \psi(0) = \Pr[L \leq 0]$, because $L \geq 0$, then $1 - \psi(0) = \Pr[L = 0]$.

Next, suppose $t_1 > 0$ is the first moment where $S(t) - ct < 0$. Then define it $L_1 = S(t_1) - ct_1$. Suppose $t_2 > t_1 > 0$ is the first moment $S(t) - ct > L_1$, Define $L_2 = S(t_2) - ct_2 - L_1$. Suppose $t_3 > t_2 > t_1 > 0$ is the first moment $S(t) - ct > L_1 + L_2$, Define $L_3 = S(t_3) - ct_3 - (L_1 + L_2)$, and so on.

Since the process is stationary, it is $L_1, L_2, \ldots, L_N$ mutually free and identically distributed, so it can be written:

$$L = L_1 + L_2 + \cdots + L_N\,.$$

2.5. Tips

The expected claim payment per unit of time for compound Poisson-distributed claims is $E[S(t)] = \lambda p_1$ where p_1 is a large average of claims, c. The average premium received is greater than the expected claim payment per unit of time, so that,

$c > \lambda p_1$, dan $c = (1+\theta)\lambda p_1$ where θ is a *Relative Security Loading* [23-24].

If the insurance company performs reinsurance, then the average reinsurance premium is determined as $C_h = (1+\xi)\lambda E[h(x)]$, where ξ is a *Reinsurance Security Loading*.

On Equation (1) *Ruin theory* is defined as

$$\psi(u) = \frac{\exp(-Ru)}{E[\exp(-RU(T)|T < \infty)]}$$

where R is the *adjudication coefficient*, which is the smallest positive number that will satisfy the equation (2).

$$M_{S(t)-ct}(r) = 1$$

Equation (2) The above can be expressed in other equations, namely:

$$\Rightarrow \; M_{S(t)-ct}(r) = 1$$
$$\Rightarrow \; E\left(e^{r(S(t)-ct)}\right) = 1$$
$$\Rightarrow \; E\left(e^{rS(t)-rct}\right) = 1$$
$$\Rightarrow \; e^{-rct}.M_{S(t)}(r) = 1$$
$$\Rightarrow \; e^{-rct}.e^{\lambda t[M_x(r)-1]} = 1$$

and then;

$$e^{-rct} = \frac{1}{e^{\lambda t[M_x(r)-1]}}$$
$$e^{-rc} = \frac{1}{e^{\lambda[M_x(r)-1]}}$$
$$\frac{1}{e^{rc}} = \frac{1}{e^{\lambda[M_x(r)-1]}}$$
$$e^{rc} = e^{\lambda[M_x(r)-1]}$$

so that Equation (3) will be obtained, namely,

$$\lambda[M_x(r) - 1] = cr$$

by substituting $c = (1+\theta)\lambda p_1$ into Equation (3) a new form of equation can be obtained, namely

$$1 + (1+\theta)p_1 r = M_x(r) \quad (11)$$

If the value of R is greater, the smaller the chance of bankruptcy, and vice versa, if the value of R is smaller, the greater the chance of bankruptcy. This trait then becomes the basis for the use of *ruin theory* in the calculation of reinsurance to minimize the chance of bankruptcy for an insurance company. This means that reinsurance is carried out with the aim of increasing the value of R.

The sum of aggregate claims that the reinsurance company depends on is defined in Equations (4) and (5). Magnitude $h(x)$ The two types of reinsurance treaty methods are as follows:

1. Proportional (*Quota Share Reinsurance*)

$$h(x) = \alpha.x \qquad ; 0 \le \alpha \le 1$$

2. Non-Proportional (*Excess of Loss Reinsurance*)
   - $h(x) = 0 \qquad ; x \le \beta$
   - $h(x) = x - \beta \quad ; x > \beta$

The sum of claims that must be paid by insurance companies that previously amounted to x, So after reinsurance is carried out, the sum of claims to be paid will be as large as $x - h(x)$ and premiums received by insurance companies that were previously equal to c, After reinsurance is carried out, it will be as large as $c - C_h$. Therefore, Equation (3) the value of the *Adjusment Coefficient* R or hereinafter referred to as R_h that is, the value of *the Adjusment Coefficient* after reinsurance can be written as Equation (6) as follows:

$$\lambda\left[M_{x-h(x)}(r) - 1\right] = (c - C_h)r$$

Or it can also be written as another equation, namely:

$$\lambda\left[M_{x-h(x)}(r) - 1\right] = (c - C_h)r$$

$$\lambda E\left[e^{r(x-h(x))}\right] = (c - C_h)r + \lambda \tag{12}$$

Thus, to find the *value of the Coeffisient Adjusment* R_h for the two types of reinsurance agreement methods are as follows:

1. Proportional (*Quota Share Reinsurance*)

By substituting $h(x) = \alpha.x$ to Equation (12) above, Obtained equations to determine values R_h from *Quota Share Reinsurance* that is:

$$\lambda E\left[e^{r(x-\alpha x)}\right] = (c - C_h)r + \lambda$$

$$\lambda \int_0^{\infty} e^{r(x-\alpha x)}.f(x)dx = (c - C_h)r + \lambda \tag{13}$$

2. Non Proportional (*Excess of Loss Reinsurance*)

By substituting $h(x) = 0$ for $x \le \beta$ and $h(x) = x - \beta$ for $x > \beta$ to Equation (12), An equation will be obtained to determine the value R_h from *Excess of Loss Reinsurance* that is:

$$\lambda \int_0^{\beta} e^{rx}.f(x)dx + \lambda \int_{\beta}^{x} e^{r\beta}.f(x)dx = (c - C_h)r + \lambda \tag{14}$$

3. Results and Discussion

Suppose the aggregate claim process $S(t)$ is a compound Poisson process with and X exponential distribution with a mean of 1 [25-26]. If the relative security loading θ is 25%, and the reinsurance security loading ξ is 40%. Determine the proportion α in the Quota Share Reinsurance method and the retention limit β in the Excess of Loss Reinsurance method to maximize the value of the adjustment coefficient R, to determine which method is more effective to use, compare the values of the two reinsurance methods.

Solution: To calculate the value of R, use Equation (12):

$$(c - C_h)r + \lambda = \lambda.E\left[e^{r(X-h(X))}\right]$$

According to the equation $c = (1+\theta)\lambda p_1$, known $E(X) = p_1 = 1$, so that is obtained:

$$\begin{aligned} c &= (1+25\%)\lambda p_1 \\ &= (1+0{,}25)\lambda \\ &= 1{,}25\lambda \end{aligned}$$

to calculate $E[h(X)]$, use the mathematical expectation with $h(X) = \alpha x$ and $f(x) = e^{-x}$, yield:

$$E[h(X)] = \int_0^\infty \alpha x \,.\, e^{-x} dx \qquad = \alpha$$

according to equation $C_h = (1+\xi)\lambda E[h(x)]$ obtained $C_h = 1{,}4\alpha\lambda$.

If the reinsurance agreement method used is Quota Share Reinsurance, then $h(x) = \alpha x$. To find the value of R_h , first find the moment generating function of $M_{x-h(x)}(r)$, i.e :

$$\begin{aligned} M_{x-h(x)}(r) &= E\left[e^{r(x-h(x))}\right] \\ &= \int_0^\infty e^{r(x-\alpha x)} . f(x) dx \end{aligned}$$

Because claims are exponentially distributed with mean 1, then $f(x) = e^{-x}$, so:

$$\begin{aligned} \mathrm{E}\left[\mathrm{e}^{\mathrm{r}(\mathrm{x}-\mathrm{h}(\mathrm{x}))}\right] &= \int_0^\infty \mathrm{e}^{\mathrm{r}(\mathrm{x}-\alpha\mathrm{x})} . \mathrm{f}(\mathrm{x}) \mathrm{dx} \\ &= \int_0^\infty \mathrm{e}^{\mathrm{r}(\mathrm{x}-\alpha\mathrm{x})} \mathrm{e}^{-\mathrm{x}} \mathrm{dx} \\ &= \frac{1}{1-(1-\alpha)\mathrm{r}} \end{aligned}$$

After obtaining the moment generating function, then substitute it into Equation (12) to find the value of $\mathrm{R_h}$,

$$\begin{aligned} \lambda \mathrm{E}\left[\mathrm{e}^{\mathrm{r}(\mathrm{x}-\alpha\mathrm{x})}\right] &= (\mathrm{c}-\mathrm{C_h})\mathrm{r} + \lambda \\ \lambda \frac{1}{1-(1-\alpha)\mathrm{r}} &= (\mathrm{c}-\mathrm{C_h})\mathrm{r} + \lambda \end{aligned}$$

Because the average (mean) = $= \lambda = 1$, then

$$\begin{aligned} \frac{1}{1-(1-\alpha)r} &= (1{,}25 - 1{,}4\alpha)r + 1 \\ r &= \frac{(0{,}25 - 0{,}4\alpha)}{(1-\alpha)(1{,}25 - 1{,}4\alpha)} \end{aligned}$$

So the Adjustment Coefficient R_h solution is:

$$\begin{aligned} R &= \frac{(0{,}25-0{,}4\alpha)}{(1-\alpha)(1{,}25-1{,}4\alpha)} \\ &= \frac{(0{,}25 - 0{,}4\alpha)}{(1{,}4\alpha^2 - 2{,}65\alpha + 1{,}25)} \end{aligned}$$

To find out the value α and for each $E[h(X)]$, consider the table below:

Table 1. Value $E[h(X)]$, proportion α and Adjustment Coefficient R for Quota Share Reinsurance

E[h(X)]	α	R	E[h(X)]	A	R	E[h(X)]	α	R
0	0	0.2	0.21	0.21	0.21979768	0.41	0.41	0.215625313
0.01	0.01	0.201039521	0.22	0.22	0.220480157	0.42	0.42	0.213563913
0.02	0.02	0.202077558	0.23	0.23	0.221115092	0.43	0.43	0.211176088
0.03	0.03	0.203113266	0.24	0.24	0.22169757	0.44	0.44	0.208427219
0.04	0.04	0.204145729	0.25	0.25	0.222222222	0.45	0.45	0.205278592
0.05	0.05	0.205173952	0.26	0.26	0.22268318	0.46	0.46	0.201686835
0.06	0.06	0.206196854	0.27	0.27	0.223074023	0.47	0.47	0.197603264
0.07	0.07	0.207213262	0.28	0.28	0.223387723	0.48	0.48	0.192973117
0.08	0.08	0.2082219	0.29	0.29	0.223616581	0.49	0.49	0.187734668
0.09	0.09	0.209221384	0.3	0.3	0.223752151	0.5	0.5	0.181818182
0.1	0.1	0.21021021	**0.31**	**0.31**	**0.223785166**	0.51	0.51	0.175144685
0.11	0.11	0.211186746	0.32	0.32	0.223705442	0.52	0.52	0.167624521
0.12	0.12	0.212149219	0.33	0.33	0.22350178	0.53	0.53	0.159155637
0.13	0.13	0.213095699	0.34	0.34	0.223161851	0.54	0.54	0.149621546
0.14	0.14	0.214024094	0.35	0.35	0.222672065	0.55	0.55	0.138888889
0.15	0.15	0.214932127	0.36	0.36	0.222017426	0.56	0.56	0.126804526
0.16	0.16	0.215817321	0.37	0.37	0.221181369	0.57	0.57	0.113192015
0.17	0.17	0.216676985	0.38	0.38	0.220145566	0.58	0.58	0.097847358
0.18	0.18	0.217508187	0.39	0.39	0.218889717	0.59	0.59	0.080533824
0.19	0.19	0.218307739	0.4	0.4	0.217391304	0.6	0.6	0.06097561
0.2	0.2	0.219072165				0.61	0.61	0.038850039

From Table 1 above it can be seen that for an $E[h(X)]$ value of 0, then the value of α is 0, and the Adjustment coefficient R value obtained is equal to 0,2 , and so on. Furthermore, to find the α value that maximizes the value of R_h the mathematical calculations, as follows:

$$\frac{\mathrm{dR}}{\mathrm{d\alpha}} = 0$$

In order to obtain the value of $\boldsymbol{\alpha = 0,941932845}$ or $\boldsymbol{= 0,308067154}$. Then substitute the values of α obtained into equation R, so:

for $= \mathbf{0,308067154}$ **,** then

$$R = \frac{(0{,}25 - 0{,}4\alpha)}{(1-\alpha)(1{,}25 - 1{,}4\alpha)} = 0{,}223787246$$

for $= \mathbf{0,941932845}$, then

$$R = \frac{(0{,}25 - 0{,}4\alpha)}{(1-\alpha)(1{,}25 - 1{,}4\alpha)} = 31{,}77621247$$

due to value R must be the smallest positive root number that satisfies the equation $\lambda[M_{x-h(x)}(r) - 1] = (c - C_h)r$, then value α taken are $\boldsymbol{\alpha = 0,308067154 \approx 0,31.}$

If using the *Excess of Loss Reinsurance method*, then the amount $h(x) = x - \beta$ for $x > \beta$, and $h(x) = 0$ for $x \leq \beta$.
To calculate the magnitude $E[h(X)]$, Use math expectations with $h(X) = x - \beta$ and $f(x) = e^{-x}$, so that,

$$\begin{aligned} E[h(X)] &= \int_{\beta}^{\infty} (x - \beta). e^{-x} dx \\ &= e^{-\beta} \end{aligned}$$

Thus, the magnitude of the retention limit value β searchable, i.e. $\beta = -\ln(E[h(X)])$.

$$\begin{aligned} C_h &= 1{,}4\lambda E[(x - \beta)] \\ &= 1{,}4\lambda e^{-\beta} \end{aligned}$$

Moment generating function of $M_{x-h(x)}(r)$, with $f(x) = e^{-x}$ are as follows:

$$\begin{aligned} M_{x-h(x)}(r) &= E\left[e^{r(x-h(x))}\right] \\ &= \frac{1 - re^{-\beta(1-r)}}{(1-r)} \end{aligned}$$

After obtaining the function of generating the moment, then substitute it into Equation (4.8) to find the value R_h,

$$(c - C_h)r + \lambda = \lambda E\left[e^{r(x-h(x))}\right]$$
$$\left(1{,}25\lambda - 1{,}4\lambda e^{-\beta}\right)r + \lambda = \lambda \frac{1 - re^{-\beta(1-r)}}{(1-r)}$$

known average (mean) $= \lambda = 1$, Then the following equation form is obtained :

$$\left(1{,}25 - 1{,}4e^{-\beta}\right)r + 1 = \frac{1 - re^{-\beta(1-r)}}{(1-r)}$$
$$1 + \left(1{,}25 - 1{,}4e^{-\beta}\right)r - \frac{1 - re^{-\beta(1-r)}}{(1-r)} = 0$$

Form equations for calculating values R It is quite complicated if done manually, so to further facilitate the calculation process, use the help of Maple software 9.5. So that values are obtained R of each value $E[h(X)]$ and retention limit values β which varies as in Table 2 below:

Table 2. Values $E[h(X)]$, Retention limits β and *Adjusment Coefficient R* for *Excess of Loss Reinsurance*

E[h(X)]	β	R	E[h(X)]	β	R	E[h(X)]	β	R
0	∞	0.2	0.21	1.5606	0.310726	0.41	0.8916	0.347497
0.01	4.6052	0.212462	0.22	1.5141	0.314319	0.42	0.8675	0.345863
0.02	3.912	0.220438	0.23	1.4697	0.317803	0.43	0.844	0.343626
0.03	3.5066	0.22729	0.24	1.4271	0.321171	0.44	0.821	0.348414
0.04	3.2189	0.233519	0.25	1.3863	0.324417	0.45	0.7985	0.33708
0.05	2.9957	0.239332	0.26	1.3471	0.32753	0.46	0.7765	0.332622
0.06	2.8134	0.24484	0.27	1.3093	0.3305	0.47	0.755	0.327257
0.07	2.6593	0.250113	0.28	1.273	0.333317	0.48	0.734	0.320887
0.08	2.5257	0.255194	0.29	1.2379	0.335968	0.49	0.7133	0.313399
0.09	2.4079	0.260113	0.3	1.204	0.338437	0.5	0.6931	0.304666
0.1	2.3026	0.264894	0.31	1.1712	0.340709	0.51	0.6733	0.294545
0.11	2.2073	0.269551	0.32	1.1394	0.342766	0.52	0.6539	0.282875
0.12	2.1203	0.274097	0.33	1.1087	0.344588	0.53	0.6349	0.26947
0.13	2.0402	0.278539	0.34	1.0788	0.346154	0.54	0.6162	0.254123
0.14	1.9661	0.282885	0.35	1.0498	0.347438	0.55	0.5978	0.236595
0.15	1.8971	0.287137	0.36	1.0217	0.348414	0.56	0.5798	0.216612
0.16	1.8326	0.291298	0.37	0.9943	0.349052	0.57	0.5621	0.193863
0.17	1.772	0.295368	**0.38**	**0.9676**	**0.349319**	0.58	0.5447	0.167986
0.18	1.7148	0.299349	0.39	0.9416	0.349176	0.59	0.5276	0.138567
0.19	1.6607	0.303237	0.4	0.9163	0.348585	0.6	0.5108	0.105124
0.2	1.6094	0.307031				0.61	0.4943	6.71E-02

From Table 2 above, it can be explained that for the value $E[h(X)]$ as big as 0, hence the magnitude β is infinite, and the value of *Adjusment coefficient R* What is obtained is as big as 0,2, meaning if retention limit β Insurance companies are infinitely large, hence the magnitude of the adjustment factor (*Adjusment coefficient*) *R* in minimizing the chances of *ruin as* large as 0,2. If $E[h(X)]$ as big as 0,01 Retention limits β insurance companies as large as 4,60517, hence the magnitude of the adjustment factor (*Adjusment coefficient*) *R* in minimizing the chances of *ruin as* large as 0,212462, and so on**.** From values *R* It gets value *R* the maximum is as large as **0.349319** i.e, for value β as big as **0.967584** and $E[h(X)]$ as big as **0,38**. Furthermore, to determine which method is more effectively used to minimize the chance of bankruptcy (*ruin*), a comparison table will be given between *the Quota Share Reinsurance method* and the *Excess of Loss Reinsurance method*.

For each value $E[h(X)]$ of the same magnitude it can be seen that the value of the *Adjusment Coefficient R* in the *Excess of Loss Reinsurance method* is always greater than the value of *the Adjusment Coefficient R* on the *Quota Share Reinsurance* method. So it can be concluded that the *Excess of Loss Reinsurance method is* more effective than the *Quota Share Reinsurance* method in minimizing the chance *of ruin* (bankruptcy) of an insurance company [27-30].

Table 3. Comparison of *Adjusment coefficient values R* for *Quota Share Reinsurance dan Excess of Loss Reinsurance*

E[h(X)]	Quota Share		Excess of Loss		E[h(X)]	Quota Share		Excess of Loss	
	A	**R**	**β**	**R**		**α**	**R**	**β**	**R**
0	0	0.2	infinite	0.2	0.31	0.31	0.2237852	1.17118	0.3407087
0.01	0.01	0.2010395	4.60517	0.2124616	0.32	0.32	0.2237054	1.13943	0.3427658
0.02	0.02	0.2020776	3.912023	0.2204376	0.33	0.33	0.2235018	1.10866	0.3445882
0.03	0.03	0.2031133	3.506558	0.2272903	0.34	0.34	0.2231619	1.07881	0.3461537
0.04	0.04	0.2041457	3.218876	0.2335187	0.35	0.35	0.2226721	1.04982	0.3474382
0.05	0.05	0.205174	2.995732	0.2393315	0.36	0.36	0.2220174	1.02165	0.3484144
0.06	0.06	0.2061969	2.813411	0.2448403	0.37	0.37	0.2211814	0.99425	0.3490523
0.07	0.07	0.2072133	2.65926	0.2501129	0.38	0.38	0.2201456	0.96758	0.3493187
0.08	0.08	0.2082219	2.525729	0.2551936	0.39	0.39	0.2188897	0.94161	0.3491765
0.09	0.09	0.2092214	2.407946	0.2601133	0.4	0.4	0.2173913	0.91629	0.3485845
0.1	0.1	0.2102102	2.302585	0.2648939	0.41	0.41	0.2156253	0.8916	0.347497
0.11	0.11	0.2111867	2.207275	0.2695512	0.42	0.42	0.2135639	0.8675	0.345863
0.12	0.12	0.2121492	2.120264	0.2740969	0.43	0.43	0.2111761	0.84397	0.3436257
0.13	0.13	0.2130957	2.040221	0.2785394	0.44	0.44	0.2084272	0.82098	0.3484144
0.14	0.14	0.2140241	1.966113	0.2828847	0.45	0.45	0.2052786	0.79851	0.3370801
0.15	0.15	0.2149321	1.89712	0.2871366	0.46	0.46	0.2016868	0.77653	0.3326216
0.16	0.16	0.2158173	1.832581	0.2912976	0.47	0.47	0.1976033	0.75502	0.3272573
0.17	0.17	0.216677	1.771957	0.2953684	0.48	0.48	0.1929731	0.73397	0.3208873
0.18	0.18	0.2175082	1.714798	0.2993488	0.49	0.49	0.1877347	0.71335	0.3133991
0.19	0.19	0.2183077	1.660731	0.3032371	0.5	0.5	0.1818182	0.69315	0.3046662
0.2	0.2	0.2190722	1.609438	0.3070309	0.51	0.51	0.1751447	0.67334	0.2945455
0.21	0.21	0.2197977	1.560648	0.3107263	0.52	0.52	0.1676245	0.65393	0.2828748
0.22	0.22	0.2204802	1.514128	0.3143188	0.53	0.53	0.1591556	0.63488	0.2694705
0.23	0.23	0.2211151	1.469676	0.3178027	0.54	0.54	0.1496215	0.61619	0.2541233
0.24	0.24	0.2216976	1.427116	0.3211712	0.55	0.55	0.1388889	0.59784	0.2365949
0.25	0.25	0.2222222	1.386294	0.3244165	0.56	0.56	0.1268045	0.57982	0.2166123
0.26	0.26	0.2226832	1.347074	0.3275296	0.57	0.57	0.113192	0.56212	0.1938627
0.27	0.27	0.223074	1.309333	0.3305004	0.58	0.58	0.0978474	0.54473	0.1679862
0.28	0.28	0.2233877	1.272966	0.3333173	0.59	0.59	0.0805338	0.52763	0.1385671
0.29	0.29	0.2236166	1.237874	0.3359676	0.6	0.6	0.0609756	0.51083	0.1051238
0.3	0.3	0.2237522	1.203973	0.3384367	0.61	0.61	0.03885	0.4943	6.71E-02

4. Conclusion

The basic principle of using ruin theory in calculating Quota Share Reinsurance and Excess of Loss Reinsurance is to calculate the maximum Adjustment Coefficient value. After doing the calculations, it is proven that the Excess of Loss Reinsurance method is more effective than the Quota Share Reinsurance method in minimizing the chances of ruin (bankruptcy) of an insurance company because, for every value that is the same size, the value of the Adjustment Coefficient in the Excess of Loss Reinsurance method is always greater than Adjustment Coefficient value on the Quota Share Reinsurance method.

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