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Article

Optimal Control with Treatment and Water Sanitation for Cholera Epidemic Model

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Abstract. This paper proposes a mathematical model for cholera using optimal control of treatment through quarantine and water sanitation. Cholera is acute diarrhoea caused by Vibrio cholera bacteria infecting the intestinal tract. The analysis related to the spread of this disease is carried out through a mathematical approach. The constructed mathematical model is demonstrated epidemiologically. The proposed optimal control is the treatment of infected individuals during the quarantine period and sanitation, namely environmental hygiene, especially water. This strategy aims to suppress the number of infected individuals and reduce the concentration of bacteria due to cholera disease. To solve the optimal control problem, we employ the 4th-order forward-backward Runge-Kutta method. Based on the simulation results, the number of individuals infected by cholera and the concentration of bacteria decreased significantly. Moreover, the proposed method can transfer infected to recovered individuals faster than an optimal control treatment.

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1. Introduction

Acute diarrhoea caused by Vibrio cholera bacteria infecting the intestinal tract is known as cholera. Cholera outbreaks could have been caused by consuming water contaminated with the bacteria, as John Snow had proven in 1854 [1]. There are several ways of transmitting this disease, one of which is due to close contact between one individual and another infected individual. However, other factors of transmission need to be investigated. Individuals who are susceptible to infection can become infected individuals if they come into contact with these individuals. Individuals at an elevated risk for infection can transmit the disease to others who cohabitate with them via storage containers for food and water [1-3].

Symptoms are not always present when an individual becomes infected. One may experience symptoms such as vomiting, leg cramps, watery diarrhoea due to this disease. Infected and untreated individuals may become dehydrated, develop acidosis and collapse. This situation can cause death within 12-24 hours [1],[4]. Several studies and experiments have shown that recovered individuals have more immunity to this disease for 3-10 years. Nevertheless, recent studies indicate that the immune system's protection may diminish after a few weeks or months [2],[5].

The SIR-type model with health education, treatment, vaccination and quarantine as a control strategy to reduce this disease was proposed by Mwasa et al., 2011 [4]. The use of quarantine measures for controlling epidemic diseases has long been a source of debate, given the political, ethical, and socioeconomic considerations involved. Achieving a balance between the greater good of the community and individual rights is critical in implementing such strategies [2],[5]. Quarantine is an action to separate people and animals that may be infected with infectious diseases [5],[6]. According to the World Health Organization (WHO), there is no suggestion to enforce quarantine measures or impose limits on individuals' mobility in response to cholera. Despite advances in medicine, cholera remains one of the diseases that can be placed under quarantine through the Emergency Use Authorization (EUA) [2],[7].

Optimal control is a method used to find the optimal way to control the dynamics of the disease-spreading system via a mathematical approach [8-10]. Several studies which have implemented optimal control of the cholera model can be found in [11-20]. The complex problem for analyzing cholera outbreaks discussing stochastic model, age-structured cholera transmission, and cholera-schistosomiasis coinfection can be observed in [21-23]. In a previous study [2],[3], a SIQRB (Susceptible-Infectious-Quarantined-Recovered-Bacterium) model was proposed, in which it was assumed that infected individuals were quarantined during the treatment period, while research conducted by Posny et al., 2015 [24] where the optimal control strategy used is water sanitation with the SVIRB (Susceptible-Vaccinated-Infectious-Recovered-Bacterium) type model modified from [25].

The primary contribution in this paper is implementing the optimal control strategies, namely a combination of treatment for infected individuals through quarantine and water sanitation in the SIQRB-type model. The proposed optimal control strategies through treatment and water sanitation hopefully can reduce infected individuals and the concentration of bacteria that cause cholera significantly.

2. Experimental Section

This section describes the steps involved in working out an optimal strategy to control the spread of cholera through water treatment and sanitation. Some of the actions taken are as follows:

2.1. Formulation of Mathematical Models and Objective Functions

In this section, we discuss the development of a mathematical SIQRB model that considers the most efficient approaches to controlling cholera by improving water sanitation and treatment methods. The approach from the structure of the objective function used in this study is the Lagrange approach.

2.2. Optimal Control Solution
The 4th-order forward-backwards Runge-Kutta method is employed to address the optimal control issue.

2.3. Numerical Simulation
In general, numerical simulations help test mathematical models constructed based on the object or problem under study. For example, numerical simulations can be used in optimal control to analyze how effective the proposed control strategy is. The simulation is carried out using the MATLAB program.

2.4. Drawing Conclusions and Suggestions
Based on the analysis of the simulation results that have been carried out, a conclusion can be drawn on the proposed strategy and suggestions for future research.

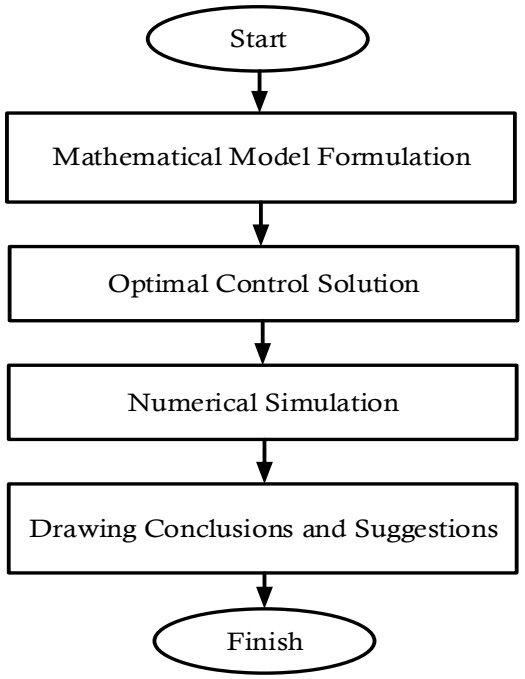

Figure 1. Research methodology

3. Results and Discussion
In this section, the dynamic model of cholera transmission in SIQRB type is modified by including treatment methods for individuals in the quarantine period and water sanitation category. The subsequent phase involves determining the optimal control approach for the model established and evaluated through numerical simulation. We utilize cholera epidemic data from the Artibonite Department of Haiti between November 2010 and May 2011 [26] as in [2].

3.1 Mathematical Model Formulation of Cholera
This section proposes a type of SIQRB model that considers the concentration of bacteria in the dynamic cholera model. At any given time $t \geq 0$, the overall population $N(t)$ is clustered into 4 clusters, namely individuals susceptible to cholera infection, $S_u(t)$, individuals infected by cholera displaying symptoms, $I_n(t)$, individuals under treatment via quarantine, $Q_u(t)$, and individuals declared recovered from cholera, $R_e(t)$. On the other hand, a cluster of bacteria, $B_a(t)$, is also

considered, reflecting the bacteria concentration at time-t. The initial conditions and parameters used in the SIQRB model can be shown in Table 1 [2], [3].

Table 1. The initial conditions and parameters of the SIQRB Model

Parameters	Descriptions	Values
ζ	Recruitment rate	$24.4N(0)/365000$
γ	Natural mortality rate	2.2493×10^{-5}
κ	Saturation constant	0.8
β	Ingestion rate	10^6
ω	Immunity rate	$0.4/365$
δ	Quarantine rate	0.05
ε	Recovery rate	0.2
γ_1	Infected individual mortality rate	0.015
γ_2	Quarantined individual mortality rate	0.0001
η	Shedding rate of infected individuals	10
α	Bacteria mortality rate	0.33
$S_u(0)$	Susceptible individuals at t = 0	5750
$I_n(0)$	Infectious individuals at t = 0	1700
$Q_u(0)$	Quarantined individuals at t = 0	0
$R_e(0)$	Recovered individuals at t = 0	0
$B_a(0)$	Bacteria concentration t = 0	275×10^3

This study proposes optimal control, namely a treatment for infected individuals by cholera through quarantine and water sanitation, into the dynamic cholera model. The results of model construction by considering this strategy can be represented by a system of nonlinear ordinary differential equations as follows:

$$\begin{aligned}
\dot{S}_u(t) &= \zeta - \frac{\beta B_a(t)}{\kappa + B_a(t)} S_u(t) + \omega R_e(t) - \gamma S_u(t), \\
\dot{I}_n(t) &= \frac{\beta B_a(t)}{\kappa + B_a(t)} S_u(t) - \delta u_1(t) I_n(t) - (\gamma_1 + \gamma) I_n(t), \\
\dot{Q}_u(t) &= \delta u_1(t) I_n(t) - (\varepsilon + \gamma_2 + \gamma) Q_u(t), \\
\dot{R}_e(t) &= \varepsilon Q_u(t) - (\omega + \gamma) R_e(t), \\
\dot{B}_a(t) &= \eta I_n(t) - \big(\alpha + u_2(t)\big) B_a(t),
\end{aligned} \tag{1}$$

where $u_1(t)$ is the treatment for cholera-infected individuals through quarantine, with $0 \le u_1(t) \le 1$ [2], and $u_2(t)$ is water sanitation, with $0 \le u_2(t) \le 0.1$ [24]. Equation (1) is a dynamical system of cholera spreading model modified from [2],[3] by considering water sanitation as the additional optimal control strategy.

3.2 Optimal Control Solution

The objective function of the optimal control issue with treatment for cholera-infected individuals through quarantine and water sanitation can be defined as follows:

$$J = \min \int_0^T [I_n(t) + B_a(t) + c_1 u_1^2(t) + c_2 u_2^2(t)]\, dt \qquad (2)$$

subject to

$$\begin{aligned}
\dot{S}_u(t) &= \zeta - \frac{\beta B_a(t)}{\kappa + B_a(t)} S_u(t) + \omega R_e(t) - \gamma S_u(t),\\
\dot{I}_n(t) &= \frac{\beta B_a(t)}{\kappa + B_a(t)} S_u(t) - \delta u_1(t) I_n(t) - \left(\gamma_1 + \gamma\right) I_n(t),\\
\dot{Q}_u(t) &= \delta u_1(t) I_n(t) - \left(\varepsilon + \gamma_2 + \gamma\right) Q_u(t),\\
\dot{R}_e(t) &= \varepsilon Q_u(t) - (\omega + \gamma) R_e(t),\\
\dot{B}_a(t) &= \eta I_n(t) - \left(\alpha + u_2(t)\right) B_a(t),
\end{aligned}$$

where, $S_u(0) = S_{u_0} \geq 0, I_n(0) = I_{n_0} \geq 0, Q_u(0) = Q_{u_0} \geq 0, R_e(0) = R_{e_0} \geq 0, B_a(0) = B_{a_0} \geq 0$, and $c_1 = \frac{W}{2}, c_2$ are positive constants.

The steps for solving optimal control can be carried out as follows [27-29]:

1. Determine Hamiltonian function

$$\begin{aligned}
\mathcal{H} = {} & I_n(t) + B_a(t) + c_1 u_1^2(t) + c_2 u_2^2(t)\\
& + \lambda_1 \left(\zeta - \frac{\beta B_a(t)}{\kappa + B_a(t)} S_u(t) + \omega R_e(t) - \gamma S_u(t) \right)\\
& + \lambda_2 \left(\frac{\beta B_a(t)}{\kappa + B_a(t)} S_u(t) - \delta u_1(t) I_n(t) - (\gamma_1 + \gamma) I_n(t) \right)\\
& + \lambda_3 \left(\delta u_1(t) I_n(t) - (\varepsilon + \gamma_2 + \gamma) Q_u(t) \right) + \lambda_4 \left(\varepsilon Q_u(t) - (\omega + \gamma) R_e(t) \right)\\
& + \lambda_5 \left(\eta I_n(t) - \left(\alpha + u_2(t)\right) B_a(t) \right)
\end{aligned}$$

2. Stationary conditions

$$\frac{\partial \mathcal{H}}{\partial u_1} = 0 \;\rightarrow\; u_1^*(t) = \frac{\delta I_n(t)(\lambda_2 - \lambda_3)}{2c_1}$$

$$\frac{\partial \mathcal{H}}{\partial u_2} = 0 \;\rightarrow\; u_2^*(t) = \frac{\lambda_5 B_a(t)}{2c_2}$$

Since $u_1(t) \in [0, 1]$ and $u_2(t) \in [0, 0.1]$ are closed sets, we obtain

$$u_1^*(t) = \min\left\{ \max\left\{ 0, \frac{\delta I_n(t)(\lambda_2 - \lambda_3)}{2c_1} \right\}, 1 \right\},$$

$$u_2^*(t) = \min\left\{ \max\left\{ 0, \frac{\lambda_5 B_a(t)}{2c_2} \right\}, 0.1 \right\},$$

so,

$$\begin{aligned}
\mathcal{H}^* = {} & I_n(t) + B_a(t) + c_1 \left(u_1^*(t)\right)^2 + c_2 \left(u_2^*(t)\right)^2\\
& + \lambda_1 \left(\zeta - \frac{\beta B_a(t)}{\kappa + B_a(t)} S_u(t) + \omega R_e(t) - \gamma S_u(t) \right)
\end{aligned}$$

$$+\lambda_2\left(\frac{\beta B_a(t)}{\kappa + B_a(t)}S_u(t) - \delta u_1^*(t)I_n(t) - (\gamma_1+\gamma)I_n(t)\right)$$
$$+\lambda_3\big(\delta u_1^*(t)I_n(t) - (\varepsilon+\gamma_2+\gamma)Q_u(t)\big)$$
$$+\lambda_4\big(\varepsilon Q_u(t) - (\omega+\gamma)R_e(t)\big) + \lambda_5\left(\eta I_n(t) - \big(\alpha + u_2^*(t)\big)B_a(t)\right)$$

3. State equations

$$\dot{S_u}^*(t) = \frac{\partial \mathcal{H}^*}{\partial \lambda_1} = \zeta - \frac{\beta B_a^*(t)}{\kappa + B_a^*(t)}S_u^*(t) + \omega R_e^*(t) - \gamma S_u^*(t)$$
$$\dot{I_n}^*(t) = \frac{\partial \mathcal{H}^*}{\partial \lambda_2} = \frac{\beta B_a^*(t)}{\kappa + B_a^*(t)}S_u^*(t) - \delta u_1^*(t)I_n^*(t) - (\gamma_1+\gamma)I_n^*(t)$$
$$\dot{Q_u}^*(t) = \frac{\partial \mathcal{H}^*}{\partial \lambda_3} = \delta u_1^*(t)I_n^*(t) - (\varepsilon+\gamma_2+\gamma)Q_u^*(t)$$
$$\dot{R_e}^*(t) = \frac{\partial \mathcal{H}^*}{\partial \lambda_4} = \varepsilon Q_u^*(t) - (\omega+\gamma)R_e^*(t)$$
$$\dot{B_a}^*(t) = \frac{\partial \mathcal{H}^*}{\partial \lambda_5} = \eta I_n^*(t) - \big(\alpha + u_2^*(t)\big)B_a^*(t)$$

4. Co-state equations

$$\dot{\lambda_1}^*(t) = -\frac{\partial \mathcal{H}^*}{\partial S_u^*} = \lambda_1^*(t)\left(\frac{\beta B_a^*(t)}{\kappa + B_a^*(t)} + \gamma\right) - \lambda_2^*(t)\left(\frac{\beta B_a^*(t)}{\kappa + B_a^*(t)}\right)$$
$$\dot{\lambda_2}^*(t) = -\frac{\partial \mathcal{H}^*}{\partial I_n^*} = -1 + \lambda_2^*(t)(\delta u_1^*(t) + \gamma_1 + \gamma) - \lambda_3^*(t)u_1^*(t) - \lambda_5^*(t)\eta$$
$$\dot{\lambda_3}^*(t) = -\frac{\partial \mathcal{H}^*}{\partial Q_u^*} = \lambda_3^*(t)(\varepsilon+\gamma_2+\gamma) - \lambda_4^*(t)\varepsilon$$
$$\dot{\lambda_4}^*(t) = -\frac{\partial \mathcal{H}^*}{\partial R_e^*} = -\lambda_1^*(t)\omega + \lambda_4^*(t)(\omega+\gamma)$$
$$\dot{\lambda_5}^*(t) = -\frac{\partial \mathcal{H}^*}{\partial B_a^*} = -1 + \lambda_1^*(t)\frac{\beta\kappa S_u^*(t)}{\big(\kappa + B_a^*(t)\big)^2} - \lambda_2^*(t)\frac{\beta\kappa S_u^*(t)}{\big(\kappa + B_a^*(t)\big)^2}$$
$$+\lambda_5^*(t)\big(\alpha + u_2^*(t)\big)$$

with transversality conditions

$$\lambda_i^*(T) = 0, i = 1,2,\cdots,5$$

3.3 Numerical Simulation

In this section, numerical simulations of optimal control problems are carried out in Equation (2) for $W = 2000$ [2] so that $c_1 = 1000$ while $c_2 = 20$ [24]. The 4th-order forward Runge-Kutta method is used to solve the state equations, while the solution to the co-state equations is used by the 4th-order backward Runge-Kutta method [28], [30]. In this simulation, the parameters used can be seen in Table 1. To compare the simulation results on individuals infected by cholera and the concentration of bacteria with and without control, the sub-model from Equation (1) can be written as follows [2],[3]:

$$\dot{S}_u(t) = \zeta - \frac{\beta B_a(t)}{\kappa + B_a(t)} S_u(t) - \gamma S_u(t),$$
$$\dot{I}_n(t) = \frac{\beta B_a(t)}{\kappa + B_a(t)} S_u(t) - (\gamma_1 + \gamma) I_n(t),$$
$$\dot{B}_a(t) = \eta I_n(t) - \alpha B_a(t),$$

where the sub-model is an interpretation of the cholera mathematical model without optimal control and quarantine which assumes that $\omega = \delta = \varepsilon = \gamma_2 = Q_u(0) = R_e(0) = 0$. A comparison of sub-model simulations, optimal control strategy with treatment (u_1) and optimal control strategy with treatment (u_1) and water sanitation (u_2) can be seen in Figure 2. In contrast, simulations for optimal controls u_1 and u_2 can be seen in Figure 3.

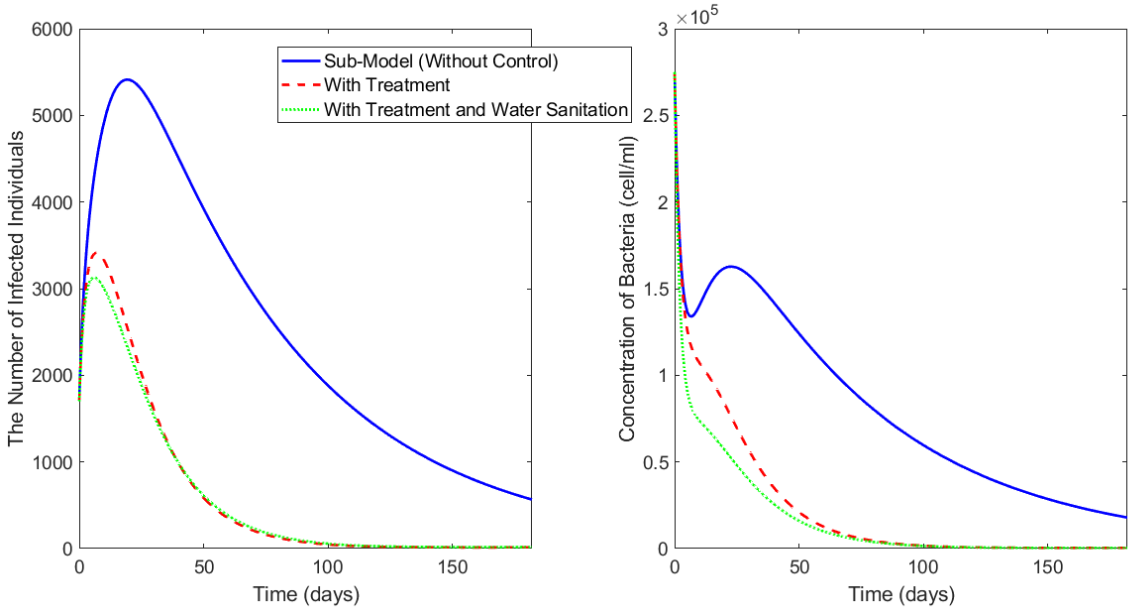

Figure 2. Optimal solutions of I_n^* and B_a^* with treatment and water sanitation strategies based on initial conditions and parameters presented in Table 1

Figure 2 shows the simulation results of the infectious individuals and the concentration of bacteria with and without control. The simulation results of the cholera sub-model (without control) and with control, namely treatment are obtained from [2], whereas the simulation results with treatment and water sanitation display our proposed strategies. For the application of one optimal control, individuals infected with cholera which was initially around 4500, became about 3300, whereas, with two optimal controls at the same time, it could suppress individuals infected by cholera which was initially about 4500 to around 3100, as shown in Figure 2.

According to Figure 3, the optimal control u_1 yields the highest possible value for $t \in [0, 110]$ days. For $t \in [110, 182]$, optimal control u_1 experiences a decline in its endpoint, whereas optimal control u_2 reaches its peak value for $t \in [0, 182]$ days. The maximum value of optimal input u_2 indicates a water sanitation strategy must be carried out continuously. Consequently, practical methods for controlling cholera, such as treatment (u_1) and water sanitation (u_2), have reduced the number of individuals affected by the disease. The difference in applying only treatment and the combination between treatment and water sanitation occurs when $t = 15$ days.

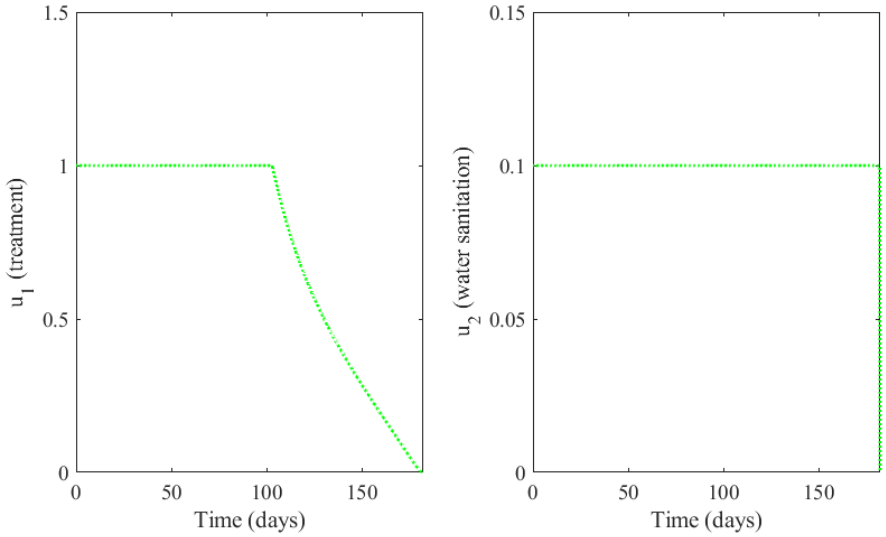

Figure 3. Optimal control u_1^* and u_2^* based on initial conditions and parameters presented in Table 1

Based on the simulation results, optimal control u_1^* and u_2^* allows a significant decrease for the number of individuals infected by cholera and the concentration of bacteria, as illustrated in Figure 2. When the controls are implemented, there is a significant drop in the highest number of individuals infected by cholera. In addition, the suggested measures to control cholera can facilitate the transfer of the disease from infected individuals to those who have recovered.

4. Conclusion

This study proposes a strategy for controlling cholera outbreaks by treating individuals infected by cholera through quarantine and water sanitation. Based on the highlighted simulation results previously, it can be concluded that applying one optimal control, namely treatment and the combination of two optimal controls, namely treatment and water sanitation, gives a significant reduction in the number of individuals infected by cholera as well as in the concentration of the bacteria that causes the disease. Therefore, the authors propose that implementing further optimal strategies, such as vaccinating susceptible individuals against cholera using the SIQRB model, could lead to a more optimal.

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