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Article

On the Metric Dimension for Snowflake Graph

Muhammad Rafif Fajri[1*], Luthfi Hadiyan Fajri[2], Jamaluddin Ashari[2], Abdurahman[3], Alifaziz Arsyad[1]

[1]Department of Mathematics and Data Science, Faculty of Mathematics and Natural Science (FMIPA), Universitas Andalas, Padang, Indonesia
[2]Department of Mathematics, Faculty of Mathematics and Natural Science (FMIPA), Universitas Negeri Padang, Padang, Indonesia
[3]Department of Electrical Engineering, Faculty of Electrical Engineering and Computer Science, Chien Hsin University of Science and Technology（健行科技大學), Taoyuan, Taiwan

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Abstract. The concept of metric dimension is derived from the resolving set of a graph, that is measure the diameter among vertices in a graph. For its usefulness in diverse fields, it is interesting to find the metric dimension of various classes of graphs. In this paper, we introduce two new graphs, namely snowflake graph and generalized snowflake graph. After we construct these graphs, aided with a lemma about the lower bound of the metric dimension on a graph that has leaves, and manually recognized the pattern, we found that dim(Snow) = 24 and dim(Snow(n,a,b,c)) = n(a+c+1).

Corresponding Author :
Muhammad Rafif Fajri
Department of Mathematics and Data Science, Faculty of Mathematics and Natural Science (FMIPA), Universitas Andalas, Padang, Indonesia
Email : rafiffajri00@gmail.com

1. Introduction

It is known that graph theory is an intense region of combinatorics that has large variety of applications in diverse fields of science. Theoretical principles of graph theory are applied to practical fields, by determining graph invariants such as vertices, edges, diameter, and degree, then using them in real-life problems. One concept that spread among all the graph theory is that of distance, and distance is used in isomorphism testing, graph operations, maximal and minimal problems on connectivity and diameter. Several parameters related to distances in graphs are highly attracting the attention of several researchers. One of them, namely, the metric dimension, has specifically centered several investigations.

Slater in [1-2] initiated the concept of a resolving set for a connected graph G (and of a minimum resolving set) under the term *location set*. He called the cardinality of a minimum resolving set the *location number* of G. Independently, Harary and Melter [3-4] introduced the same concept but used the term *metric dimension*, rather than location number.

Gerey and Johnson showed that determining the metric dimension of a graph is an NP-complete problem that is reduced from 3-dimensional marching (3DM), while Khuller et al proved that it is reduced from 3 satisfiability (3SAT). However, Chartrand et al. [5-6] have obtained some results as follows.

Theorem 1.1. [3] Let G be a connected graph of order $n \geq 2$. Then
1. $\dim(G) = 1$ if only if $G = P_n$.
2. $\dim(G) = n - 1$ if only if $G = K_n$.
3. For $n \geq 3$, $\dim(C_n) = 2$
4. $\dim(G) = n - 2$ if only if G is either $K_{r,s}$ for $r, s \geq 1$, or $K_r + \overline{K}_s$ for $r > 1$, $s \geq 2$, or $K_r + (K_1 \cup K_s)$ for $r, s \geq 1$.

Some results for certain class of graphs have been obtained, such as cycles, trees, fans, wheels, complete n-partite graphs, unicyclic graphs, grids, honeycomb networks. circulant networks, Cayley graphs, graphs with pendants, Jahangir graphs, amalgamation of cycles, regular bipartite graphs, corona product of Graphs, lexicographic product of graphs, barycentric subdivision of Cayley graphs, windmill graphs, 2-connected graph [7-9], fullerene graphs, oriented graphs, generalized wheels [10-12], binary products, cartesian powers of a graph, and Generalized Petersen graphs [13-15].

The concept of metric dimension has been developed. There are several developments obtained such as edge metric dimensions and local metric dimensions [16-17]. Some results for edge metric dimension have been acquired, such as 2-connected graphs, one-heptagonal carbon nanocone [16], hierarchical product [18-20], cacti, unicyclic graph [21-22], Generalized Petersen graphs, and some graph operation [23-24]. While, some results for local metric dimension have been acquired, such as lexicographic product of graphs, generalized hierarchical products, and graphs with small clique numbers [25-27].

Other developments related to the concept of metric dimensions are truncated metric dimensions, mixed metric dimension [28-29], strong metric dimension [30-32]], metric dimensions vs. cyclomatic number of graphs , fractional edge dimension of graphs, learning to compute metric dimensions [33-35], the dominant metric dimension, the complement metric dimension [36-37], deletion on the edge metric dimension, extremal results of bounded metric dimension, and robustness metric dimension [38-40].

A snowflake is a single ice crystal or collection of ice crystals that fall from the Earth's atmosphere. Snowflakes come in various sizes and shapes. Snowflakes are white even though they are made of pure ice. This is due to the diffuse reflection of the entire light spectrum on the small crystal. In Figure 1. An example of a snowflake is given [41-42].

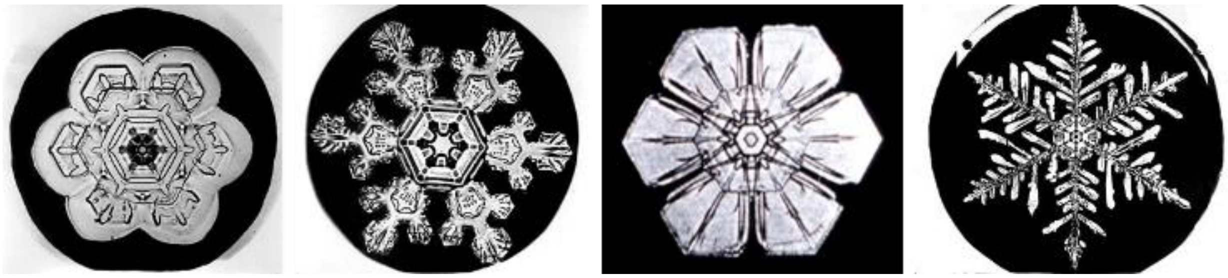

Figure 1. Photo of snowflakes by Wilson Bentley (1865–1931)

In this paper, we define some new graphs, that is a snowflake graph and a generalized snowflake graph, which are constructed by vertices and edges resembling one of the snowflake shapes. Then, we will determine the metric dimensions of the snowflake graph and the general snowflake graph.

2. Notation

The vertices and edges set of a graph G are denoted by V(G) and E(G), respectively. We refer to the general graph theory notations and terminologies that are not described in this paper to the book Graphs and Digraphs [43-44]. The number of vertices adjacent to a vertex v is called degree of vertex v. A vertex of degree one is referred to as a leaf. An edge incident with an end-vertex is called a pendant edge.

The distance $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest $u - v$ path in G. For an ordered set $W = \{w_1, w_2, \ldots, w_k\} \subseteq V(G)$ of vertices, we refer to the ordered k-tuple $r(v|W) = (d(v, w_1), d(v, w_2), \ldots, d(v, w_k))$ as the (metric) representation of v with respect to W. The set W is called a resolving set for G if $r(u|W) = r(v|W)$ implies that $u = v$, for all $u, v \in G$. A resolving set of minimum cardinality for a graph G is called a minimum resolving set or a basis for G. The metric dimension $\dim(G)$ is the number of vertices in a basis for G.

3. Results and Discussion

In this section, we define a snowflake graph and a generalized snowflake graph, and then determine the metric dimensions of these graphs. To determine such metric dimensions, we obtain a lemma related to the lower bounds of the metric dimension on a graph that contains leaves. This lemma arose from the need of ensuring that every leaf adjacent to the same vertex must have distinct representation.

Lemma 3.2. Let G be any connected graph that has n vertices that are connected to pendant edges. If $v_i \in G$ connected with k_i many vertices of degree one, then $\dim(G) \geq \sum_{i=1}^{n} (k_i - 1)$, for all $i \in [1, n]$.

Proof. We will show that $\dim(G) \geq \sum_{i=1}^{n} (k_i - 1)$, for all $i \in [1, n]$. Let G is a connected graph and vertex $v_i \in G$ connected with k_i many vertices of degree one. Suppose the vertices of degree one connected with vertex v_i are $v_{1,j}$, where $i \in [1, n]$ and $j \in [1, k_i]$. Without loss of generality, consider vertex v_1 and $v_{1,j}$. Suppose the set $W = v(G) - \{v_1, v_{1,j}\}$ is a resolving set for G. Since $d(v_1, v_{1,k_1}) = 1$ and edge $v_1 v_{1,k_1}$ is the only edge connecting v_1 and v_{1,k_i}, then representation of $v_{1,j}$ with respect to W is $r(v_{1,1}|W) = r(v_{1,2}|W) = \ldots = r(v_{1,k_1}|W) = r(v_i|W) + (\bar{1})$. So, the representation $r(v_{1,i}) \neq r(v_{1,j})$ for $i, j \in [1, k_1]$, and then any $k_1 - 1$ edges of v_{1,k_1} must be a member of resolving set W. As a result, in general, $\dim(G) \geq \sum_{i=1}^{n} (k_i - 1)$.

Q.E.D

Here we formally introduce our new graph. A snowflake graph is a graph obtained by resembling one of the snowflake shapes into vertices and edges so that it forms a simple graph denoted snowflake graph. The structure of the snowflake graph consists of tree graph and circle graph as shown in Figure 2. which is defined by the set of vertices and edges as follows.

Definition 3.3.
$V(Snow) = \{v_0, v_i, v_{i,1,k,l}, v_{i,2,k}, v_{i,3,1,l}, v_{i,k5} | 1 \leq i \leq 6; 1 \leq k \leq 2; 0 \leq l \leq 2.\}$ and
$E(Snow) = \{v_0 v_{i,3,1,0}, v_{i,3,1,0} v_{i,3,1,j}, v_{i,3,1,0} v_{i,2,2}, v_{i,2,2} v_{i,2,1}, v_{i,2,1} v_{i,1,2,0}, v_{i,1,2,0} v_{i,1,2,j}, v_{i,1,2,0} v_{i,1,1,0}, v_{i,1,1,0} v_{i,1,1,j}, v_{i,1,1,0} v_a, v_{i,2,j} v_{i,j}, v_{i,j} v_{i+1(\text{mod}6),2,j} | 1 \leq i \leq 6; 1 \leq j \leq 2\}$.

Theorem 3.4. If $Snow$ is a graph that is defined in Definition 3.3, then $dim(Snow) = 24$.

Proof. **Upper bound :**

First, we will show that $\dim(Snow) \leq 24$. Suppose an ordered set
$W = \{v_a, v_{1,1,1,1}, v_{1,1,2,1}, v_{1,3,1,1}, v_b, v_{2,1,1,1}, v_{2,1,2,1}, v_{2,3,1,1}, v_c, v_{3,1,1,1}, v_{3,1,2,1}, v_{3,3,1,1}, v_d, v_{4,1,1,1}, v_{4,1,2,1}, v_{4,3,1,1}, v_e, v_{5,1,1,1}, v_{5,1,2,1}, v_{5,3,1,1}, v_f, v_{6,1,1,1}, v_{6,1,2,1}, v_{6,3,1,1}\}$,
where $W \subseteq V(Snow)$ like figure 3.

Define the representation of $V(Snow)$ with respect to W as follow

$$r(v|W) = (d(v, v_1), d(v, v_{1,1,1,1}), d(v, v_{1,1,2,1}), d(v, v_{1,3,1,1}), d(v, v_2), d(v, v_{2,1,1,1}), d(v, v_{2,1,2,1}), d(v, v_{2,3,1,1}), d(v, v_3), d(v, v_{3,1,1,1}), d(v, v_{3,1,2,1}), d(v, v_{3,3,1,1}) d(v, v_4), d(v, v_{4,1,1,1}), d(v, v_{4,1,2,1}), d(v, v_{4,3,1,1}), d(v, v_5), d(v, v_{5,1,1,1}), d(v, v_{5,1,2,1}), d(v, v_{5,3,1,1}), d(v, v_6), d(v, v_{6,1,1,1}), d(v, v_{6,1,2,1}), d(v, v_{6,3,1,1})).$$

for all $v \in V(snow)$.

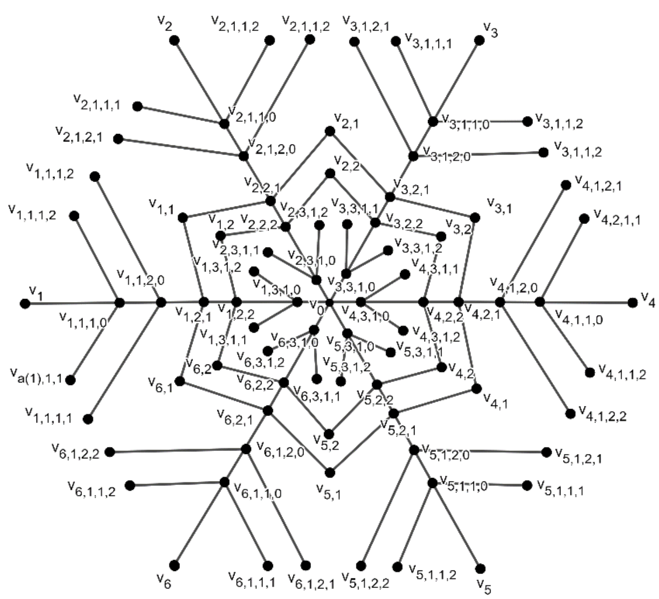

Figure 2. *Snow* graph

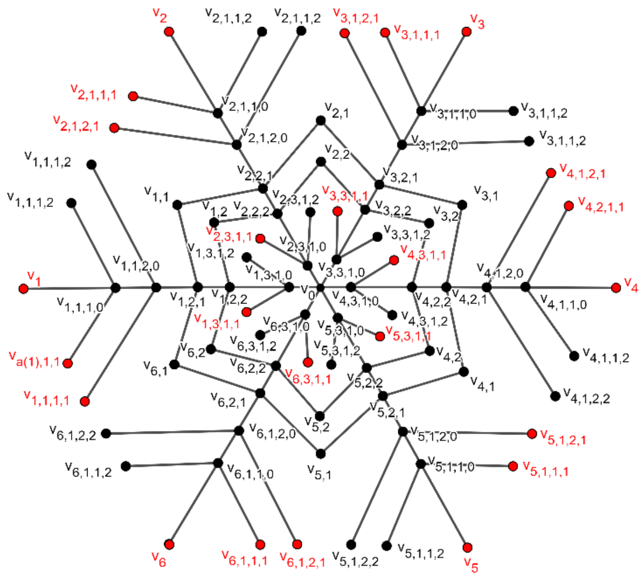

Figure 3. Partition of *snow* graph

The representation of each vertex in *Snow* is given as follows.

$r(v_1|W) = (0,2,3,6,8,8,7,8,10,10,9,8,12,12,11,8,10,10,9,8,8,8,7,8)$
$r(v_{1,1,1,0}|W) = (1,1,2,5,7,7,6,7,9,9,8,7,11,11,10,7,9,9,8,7,7,7,6,7)$
$r(v_{1,1,1,1}|W) = (2,0,3,6,8,8,7,8,10,10,9,8,12,12,11,8,10,10,9,8,8,8,7,8)$
$r(v_{1,1,1,2}|W) = (2,2,3,6,8,8,7,8,10,10,9,8,12,12,11,8,10,10,9,8,8,8,7,8)$
$r(v_{1,1,2,0}|W) = (2,2,1,4,6,6,5,6,8,8,7,6,10,10,9,6,8,8,7,6,6,6,5,6)$
$r(v_{1,1,2,1}|W) = (3,3,0,5,7,7,6,7,9,9,8,7,11,11,10,7,9,9,8,7,7,7,6,7)$
$r(v_{1,1,2,2}|W) = (3,3,2,5,7,7,6,7,9,9,8,7,11,11,10,7,9,9,8,7,7,7,6,7)$
$r(v_{1,2,1}|W) = (3,3,2,3,5,5,4,5,7,7,6,5,9,9,8,5,7,7,6,5,5,5,4,5)$
$r(v_{1,2,2}|W) = (4,4,3,2,6,6,5,4,8,8,7,4,8,8,7,4,8,8,7,4,6,6,5,4)$
$r(v_{1,1}|W) = (4,4,3,4,4,4,3,4,6,6,5,6,8,8,7,6,8,8,7,6,6,6,5,6)$
$r(v_{1,2}|W) = (5,5,4,4,5,5,4,3,7,7,6,5,9,9,8,5,9,9,8,5,7,7,6,5)$
$r(v_{1,3,1,0}|W) = (5,5,4,1,7,7,6,3,7,7,6,3,7,7,6,3,7,7,6,3,7,7,6,3)$
$r(v_{1,3,1,1}|W) = (6,6,5,0,8,8,7,4,8,8,7,4,8,8,7,4,8,8,7,4,8,8,7,4)$
$r(v_{1,3,1,2}|W) = (6,6,5,2,8,8,7,4,8,8,7,4,8,8,7,4,8,8,7,4,8,8,7,4)$
$r(v_2|W) = (8,8,7,8,0,2,3,6,8,8,7,8,10,10,9,8,12,12,11,8,10,10,9,8)$
$r(v_{2,1,1,0}|W) = (7,7,6,7,1,1,2,5,7,7,6,7,9,9,8,7,11,11,10,7,9,9,8,7)$
$r(v_{2,1,1,1}|W) = (8,8,7,8,2,0,3,6,8,8,7,8,10,10,9,8,12,12,11,8,10,10,9,8)$
$r(v_{2,1,1,2}|W) = (8,8,7,8,2,2,3,6,8,8,7,8,10,10,9,8,12,12,11,8,10,10,9,8)$
$r(v_{2,1,2,0}|W) = (6,6,5,6,2,2,1,4,6,6,5,6,8,8,7,6,10,10,9,6,8,8,7,6)$
$r(v_{2,1,2,1}|W) = (7,7,6,7,3,3,0,5,7,7,6,7,9,9,8,7,11,11,10,7,9,9,8,7)$
$r(v_{2,1,2,2}|W) = (7,7,6,7,3,3,2,5,7,7,6,7,9,9,8,7,11,11,10,7,9,9,8,7)$
$r(v_{2,2,1}|W) = (5,5,4,5,3,3,2,3,5,5,4,5,7,7,6,5,9,9,8,5,7,7,6,5)$
$r(v_{2,2,2}|W) = (6,6,5,4,4,4,3,2,6,6,5,4,8,8,7,4,8,8,7,4,8,8,7,4)$
$r(v_{2,1}|W) = (6,6,5,6,4,4,3,4,4,4,3,4,6,6,5,6,8,8,7,6,8,8,7,6)$
$r(v_{2,2}|W) = (7,7,6,5,5,5,4,4,5,5,4,3,7,7,6,5,9,9,8,5,9,9,8,5)$
$r(v_{2,3,1,0}|W) = (7,7,6,3,5,5,4,1,7,7,6,3,7,7,6,3,7,7,6,3,7,7,6,3)$

$r(v_{2,3,1,1}|W) = (8,8,7,4,6,6,5,0,8,8,7,4,8,8,7,4,8,8,7,4,8,8,7,4)$
$r(v_{2,3,1,2}|W) = (8,8,7,4,6,6,5,2,8,8,7,4,8,8,7,4,8,8,7,4,8,8,7,4)$
$r(v_3|W) = (10,10,9,8,8,8,7,8,0,2,3,6,8,8,7,8,10,10,9,8,12,12,11,8)$
$r(v_{3,1,1,0}|W) = (9,9,8,7,7,7,6,7,1,1,2,5,7,7,6,7,9,9,8,7,11,11,10,7)$
$r(v_{3,1,1,1}|W) = (10,10,9,8,8,8,7,8,2,0,3,6,8,8,7,8,10,10,9,8,12,12,11,8)$
$r(v_{3,1,1,2}|W) = (10,10,9,8,8,8,7,8,2,2,3,6,8,8,7,8,10,10,9,8,12,12,11,8)$
$r(v_{3,1,2,0}|W) = (8,8,7,6,6,6,5,6,2,2,1,4,6,6,5,6,8,8,7,6,10,10,9,6)$
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$r(v_{3,1}|W) = (8,8,7,6,6,6,5,6,4,4,3,4,4,4,3,4,6,6,5,6,8,8,7,6)$
$r(v_{3,2}|W) = (9,9,8,5,7,7,6,5,5,5,4,4,5,5,4,3,7,7,6,5,9,9,8,5)$
$r(v_{3,3,1,0}|W) = (7,7,6,3,7,7,6,3,5,5,4,1,7,7,6,3,7,7,6,3,7,7,6,3)$
$r(v_{3,3,1,1}|W) = (8,8,7,4,8,8,7,4,6,6,5,0,8,8,7,4,8,8,7,4,8,8,7,4)$
$r(v_{3,3,1,2}|W) = (8,8,7,4,8,8,7,4,6,6,5,2,8,8,7,4,8,8,7,4,8,8,7,4)$
$r(v_4|W) = (12,12,11,8,10,10,9,8,8,8,7,8,0,2,3,6,8,8,7,8,10,10,9,8)$
$r(v_{4,1,1,0}|W) = (11,11,10,7,9,9,8,7,7,7,6,7,1,1,2,5,7,7,6,7,9,9,8,7)$
$r(v_{4,1,1,1}|W) = (12,12,11,8,10,10,9,8,8,8,7,8,2,0,3,6,8,8,7,8,10,10,9,8)$
$r(v_{4,1,1,2}|W) = (12,12,11,8,10,10,9,8,8,8,7,8,2,2,3,6,8,8,7,8,10,10,9,8)$
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$r(v_{4,1}|W) = (8,8,7,6,8,8,7,6,6,6,5,6,4,4,3,4,4,4,3,4,6,6,5,6)$
$r(v_{4,2}|W) = (9,9,8,5,9,9,8,5,7,7,6,5,5,5,4,4,5,5,4,3,7,7,6,5)$
$r(v_{4,3,1,0}|W) = (7,7,6,3,7,7,6,3,7,7,6,3,5,5,4,1,7,7,6,3,7,7,6,3)$
$r(v_{4,3,1,1}|W) = (8,8,7,4,8,8,7,4,8,8,7,4,6,6,5,0,8,8,7,4,8,8,7,4)$
$r(v_{4,3,1,2}|W) = (8,8,7,4,8,8,7,4,8,8,7,4,6,6,5,2,8,8,7,4,8,8,7,4)$
$r(v_5|W) = (10,10,9,8,12,12,11,8,10,10,9,8,8,8,7,8,0,2,3,6,8,8,7,8)$
$r(v_{5,1,1,0}|W) = (9,9,8,7,11,11,10,7,9,9,8,7,7,7,6,7,1,1,2,5,7,7,6,7)$
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$r(v_{5,1}|W) = (6,6,5,6,8,8,7,6,8,8,7,6,6,6,5,6,4,4,3,4,4,4,3,4)$
$r(v_{5,2}|W) = (7,7,6,5,9,9,8,5,9,9,8,5,7,7,6,5,5,5,4,4,5,5,4,3)$
$r(v_{5,3,1,0}|W) = (7,7,6,3,7,7,6,3,7,7,6,3,7,7,6,3,5,5,4,1,7,7,6,3)$
$r(v_{5,3,1,1}|W) = (8,8,7,4,8,8,7,4,8,8,7,4,8,8,7,4,6,6,5,0,8,8,7,4)$
$r(v_{5,3,1,2}|W) = (8,8,7,4,8,8,7,4,8,8,7,4,8,8,7,4,6,6,5,2,8,8,7,4)$
$r(v_6|W) = (6,8,8,7,8,10,10,9,8,12,12,11,8,10,10,9,8,8,8,7,8,0,2,3)$

$r(v_{6,1,1,0}|W) = (7,7,6,7,9,9,8,7,11,11,10,7,9,9,8,7,7,7,6,7,1,1,2,5)$
$r(v_{6,1,1,1}|W) = (8,8,7,8,10,10,9,8,12,12,11,8,10,10,9,8,8,8,7,8,2,0,3,6)$
$r(v_{6,1,1,2}|W) = (8,8,7,8,10,10,9,8,12,12,11,8,10,10,9,8,8,8,7,8,2,2,3,6)$
$r(v_{6,1,2,0}|W) = (6,6,5,6,8,8,7,6,10,10,9,6,8,8,7,6,6,6,5,6,2,2,1,4)$
$r(v_{6,1,2,1}|W) = (7,7,6,7,9,9,8,7,11,11,10,7,9,9,8,7,7,7,6,7,3,3,0,5)$
$r(v_{6,1,2,2}|W) = (7,7,6,7,9,9,8,7,11,11,10,7,9,9,8,7,7,7,6,7,3,3,2,5)$
$r(v_{6,2,1}|W) = (5,5,4,5,7,7,6,5,9,9,8,5,7,7,6,5,5,5,4,5,3,3,2,3)$
$r(v_{6,2,2}|W) = (6,6,5,4,8,8,7,4,8,8,7,4,8,8,7,4,6,6,5,4,4,4,3,2)$
$r(v_{6,1}|W) = (4,4,3,4,6,6,5,6,8,8,7,6,8,8,7,6,6,6,5,6,4,4,3,4)$
$r(v_{6,2}|W) = (5,5,4,3,7,7,6,5,9,9,8,5,9,9,8,5,7,7,6,5,5,5,4,4)$
$r(v_{6,3,1,0}|W) = (7,7,6,3,7,7,6,3,7,7,6,3,7,7,6,3,7,7,6,3,5,5,4,1)$
$r(v_{6,3,1,1}|W) = (8,8,7,4,8,8,7,4,8,8,7,4,8,8,7,4,8,8,7,4,6,6,5,0)$
$r(v_{6,3,1,2}|W) = (8,8,7,4,8,8,7,4,8,8,7,4,8,8,7,4,8,8,7,4,6,6,5,2)$

Since $r(v_a|W) \neq r(v_b|W)$ implies that $v_a = v_b$, for all $v_a, v_b \in V(Snow)$, then W is resolving set for $Snow$. Therefore, $dim(Snow) \leq 24$.

Lower bound :

Second, we will show that $dim(Snow) \geq 24$. Choose the set edges of $A = \{v_{i,1,1,0}|i \in [1,6]$ and $v_{i,1,1,0} \in E(Snow)\}$ dan $B = \{v_{j,1,2,0}, v_{j,3,1,0}|j \in [1,6]$ �f and $\in ESnow)\}$. The sets A and B are, respectively, the set and edges connected by 3 and 2 edges of degree one, where $|A| = 6$ and $|B| = 12$. By Lemma 3.2, $dim(Snow) \geq \sum_{i=1}^{|A|} (3-1) + \sum_{i=1}^{|B|} (2-1) = 6(2) + 12 = 24$, so $dim(Snow) \geq 24$.

Putting the bounds together gives us, that $dim(Snow) = 24$.

Q.E.D

A generalized snowflake graph, denoted by $Snow(n, a, b, c)$, is a $Snow$ graph with n parts of the stem, a pair of outer leaves, b middle circle, and c pairs of inner leaves as shown in Figure 4. The set of vertices and edges of the graph is defined as follows.

Definition 3.5.

$V(Snow(n,a,b,c)) = \{v_0, v_i, v_{i,1,j,k}, v_{i,2,l}, v_{i,3,p,k}, v_{i,l}|1 \leq i \leq n; 1 \leq j \leq a; 0 \leq k \leq 2; 1 \leq l \leq b; 1 \leq p \leq c\}$ and

$E(Snow(n,a,b,c)) = \{v_0v_{i,3,c,0}, v_{i,3,j,0}v_{i,3,j+1,0}, v_{i,3,k,0}v_{i,3,k,l}, v_{i,3,1,0}v_{i,2,b}, v_{i,2,p}v_{i,2,p+1}, v_{i,2,1}v_{i,1,a,0}, v_{i,1,q,0}v_{i,1,1,0}, v_i, v_{i,2,r}v_{i,r}, v_{i,r}v_{i+1(\mathrm{mod}n),2,r}|1 \leq i \leq n; 1 \leq j \leq c-1; 1 \leq k \leq c; 1 \leq l \leq 2; 1 \leq p \leq b-1; 1 \leq q \leq a-1; 1 \leq r \leq b.\}$

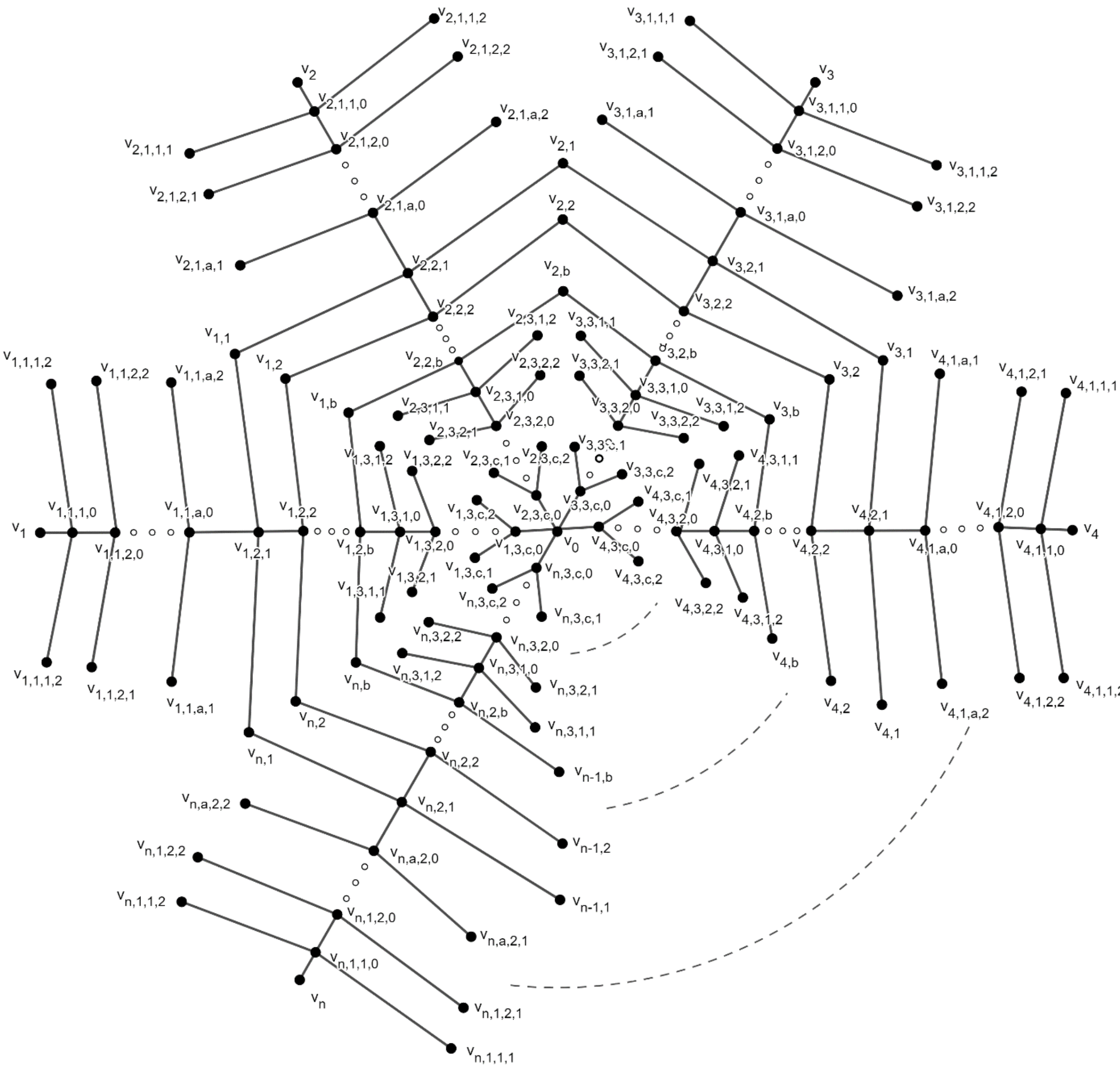

Figure 4. $Snow(n, a, b, c)$

Theorem 3.6. If $Snow(n, a, b, c)$ is a graph defined in Definition 3.5, then $dim(Snow) = n(a + c + 1)$ for all $n, a, b, c \in \mathbb{N}$ and $n \geq 3$.

Proof. **Upper Bound :**

First, we will show that $dim(\text{Snow}(n, a, b, c)) \leq n(a + c + 1)$. Suppose an ordered set

$$W = \{v_1, v_{1,1,1,1}, v_{1,1,2,1}, \dots, v_{1,1,a,1}, v_{1,3,1,1}, v_{1,3,2,1}, \dots, v_{2,3,c,1}, v_2, v_{2,1,1,1}, v_{2,1,2,1}, \dots, v_{2,1,a,1}, v_{2,3,1,1}, v_{2,3,2,1}, \dots, v_{2,3,c,1}, \dots, \dots, v_n, v_{n,1,1,1}, v_{n,1,2,1}, \dots, v_{n,1,a,1}, v_{n,3,1,1}, v_{n,3,2,1}, \dots, v_{n,3,c,1}\},$$

Where $W \subseteq V(Snow(n, a, b, c))$ and $|W| = n(a + c + 1)$.

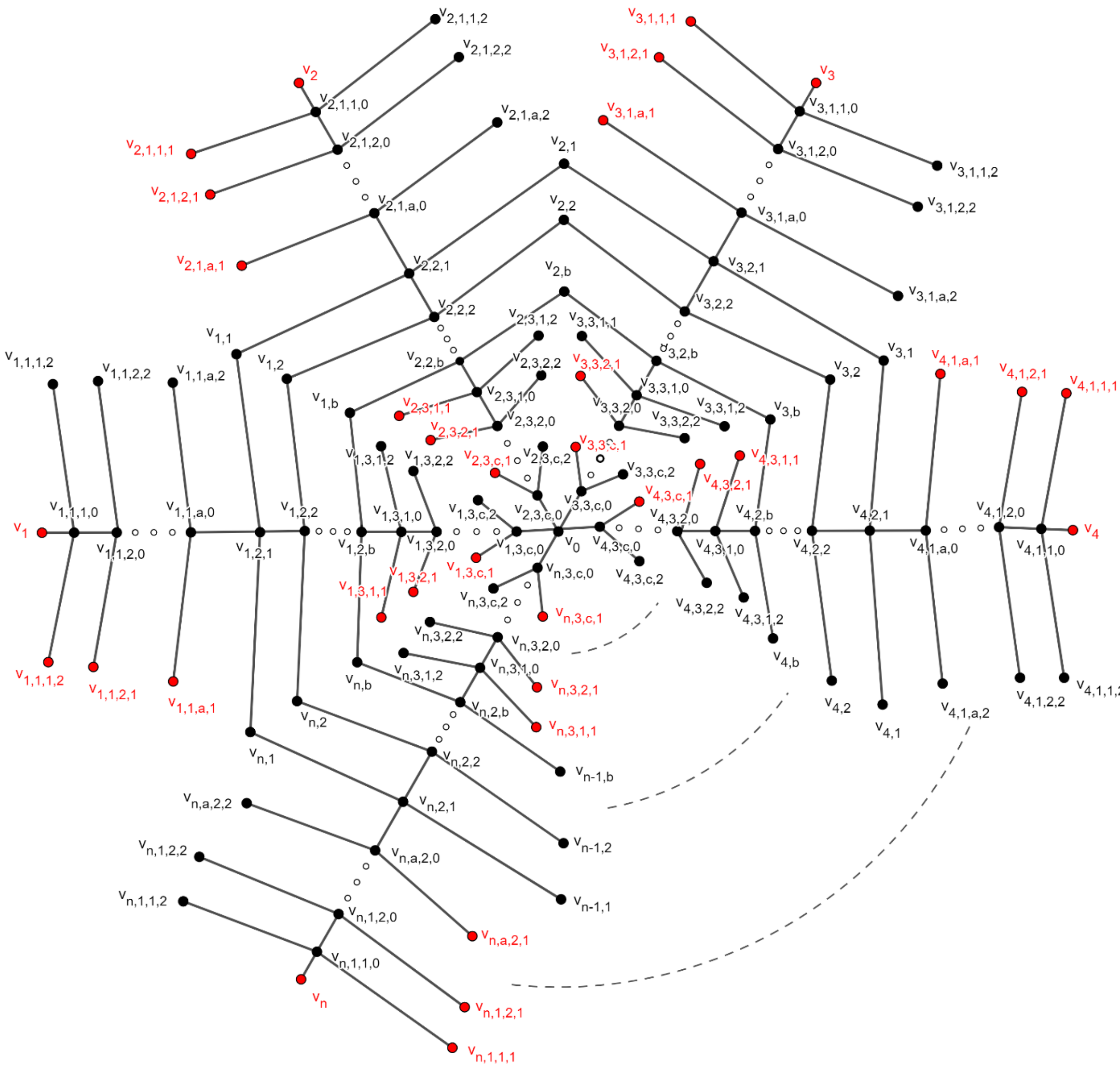

Figure 4. Partition of $snow(n, a, b, c)$ graph

Define the representation of $V(Snow(n, a, b, c))$ with respect to W as follow

$$r(v|W) = (d(v, v_1), d(v, v_{1,1,1,1}), d(v, v_{1,1,2,1}), \dots, d(v, v_{1,1,a,1}), d(v, v_{1,3,1,1}), d(v, v_{1,3,2,1}), \dots,$$
$$d(v, v_{2,3,c,1}), d(v, v_2), d(v, v_{2,1,1,1}), d(v, v_{2,1,2,1}), \dots, d(v, v_{2,1,a,1}), d(v, v_{2,3,1,1}),$$
$$d(v, v_{2,3,2,1}), \dots, d(v, v_{2,3,c,1}), \dots, \dots, d(v, v_n), d(v, v_{n,1,1,1}), d(v, v_{n,1,2,1}), \dots,$$
$$d(v, v_{n,1,a,1}), d(v, v_{n,3,1,1}), d(v, v_{n,3,2,1}), \dots, d(v, v_{n,3,c,1})),$$

for all $v \in V(Snow(n, a, b, c))$.

Let $W_1 = \{v_1, v_2, \dots, v_n\}$, $W_2 = \{v_1, v_{1,1,j,1}, v_2, v_{2,1,j,1}, \dots, v_n, v_{n,1,j,1}\}$, and $W_3 = \{v_1, v_{1,3,j,1}, v_2, v_{2,3,j,1} \dots, v_n, v_{n,3,j,1}\}$, be an ordered sets, where $W_1 \subseteq W_2 \subseteq W$ and $W_1 \subseteq W_3 \subseteq W$. So,

$r(v|W_1) = (d(v,v_1), d(v,v_2), \ldots, d(v,v_n))$, $W_2 = \{v_1, v_{1,1,j,1}, v_2, v_{2,1,j,1}, \ldots, v_n, v_{n,1,j,1}\}$, and $W_3 = \{v_1, v_{1,3,j,1}, v_2, v_{2,3,j,1} \ldots, v_n, v_{n,3,j,1}\}$.

Representation of each vertex in $Snow(n,a,b,c)$ is given as follows.

1. Representation of v_0 with respect to $W_1 \subseteq W$ is
$$r(v_0|W_1) = (a+b+c+1, a+b+c+1, \ldots, a+b+c+1).$$
2. Representation of v_i, $i \in [1,n]$, with respect to $W_1 \subseteq W$ are

$$r(v_1, W_2) = \left(0, \min\{2a+4, 2(a+b+c+1)\}, \min\{2a+6, 2(a+b+c+1)\}, \ldots, \min\left\{2\left(a+1+\left\lfloor\frac{n}{2}\right\rfloor\right), 2(a+b+c+1)\right\}, \ldots, \min\{2a+6, 2a+4, 2(a+b+c+1)\}, \min\{2a+4, 2(a+b+c+1)\}\right)$$

$$r(v_2, W_2) = \left(\min\{2a+4, 2(a+b+c+1)\}, 0, \min\{2a+4, 2(a+b+c+1)\}, \min\{2a+6, 2(a+b+c+1)\}, \ldots, \min\left\{2\left(a+1+\left\lfloor\frac{n}{2}\right\rfloor\right), 2(a+b+c+1)\right\}, \ldots, \min\{2a+6, 2a+4, 2(a+b+c+1)\}\right)$$

$$\vdots$$

$$r(v_n, W_2) = \left(min\{2a+4, 2(a+b+c+1)\}, min\{2a+6, 2(a+b+c+1)\}, \ldots, min\left\{2\left(a+1+\left\lfloor\frac{n}{2}\right\rfloor\right), 2(a+b+c+1)\right\}, \ldots, min\{2a+6, 2a+4, 2(a+b+c+1)\}, min\{2a+4, 2(a+b+c+1)\}, 0\right)$$

Since $r(v_r, W_1) \neq r(v_s, W_1)$ and $W_1 \subseteq W$, then $r(v_r, W) \neq r(v_s, W)$, for all $r, s \in [1,n]$.

3. Representation $v_{i,1,j,0}$, $i \in [1,n]$ and $j \in [1,a]$ with respect to $W_1 \subseteq W$ are

$$r(v_{1,1,j,0}, W_1) = \left(j, \min\{2a+4-j, 2(a+b+c+1)-j\}, \min\{2a+6-j, 2(a+b+c+1)-j\}, \ldots, \min\left\{2\left(a+1+\left\lfloor\frac{n}{2}\right\rfloor\right)-j, 2(a+b+c+1)-j\right\}, \ldots, \min\{2a+6-j, 2(a+b+c+1)-j\}, \min\{2a+4-j, 2(a+b+c+1)-j\}\right\}$$

$$r(v_{1,1,j,0}, W_1) = \left(\min\{2a+4-j, 2(a+b+c+1)-j\}, j, \min\{2a+4-j, 2(a+b+c+1)-j\}, \min\{2a+6-j, 2(a+b+c+1)-j\}, \ldots, \min\left\{2\left(a+1+\left\lfloor\frac{n}{2}\right\rfloor\right)-j, 2(a+b+c+1)-j\right\}, \ldots, \min\{2a+6-j, 2(a+b+c+1)-j\}\right)$$

$$\vdots$$

$$r(v_{1,1,j,0}, W_1) = \left(\min\{2a+4-j, 2(a+b+c+1)-j\}, \min\{2a+6-j, 2(a+b+c+1)-j\}, \ldots, \min\left\{2\left(a+1+\left\lfloor\frac{n}{2}\right\rfloor\right)-j, 2(a+b+c+1)-j\right\}, \ldots, \min\{2a+6-j, 2(a+b+c+1)-j\}, \min\{2a+4-j, 2(a+b+c+1)-j\}, j\right)$$

Since $r(v_{r,1,j,0}, W_1) \neq r(v_{s,1,j,0}, W_1)$ and $W_1 \subseteq W$, then $r(v_{r,1,j,0}, W) \neq r(v_{s,1,j,0}, W)$, for all $r, s \in [1,n]$.

4. Representation $v_{i,1,j,k}$, $\forall i \in [1,n]$, $j \in [1,a]$, and $k \in [1,2]$ with respect to $W_2 \subseteq W$ as follow

$$r(v_{1,1,j,1}|W_2) = \left(\mathrm{j}+1, 0, \min\{2a-j+5, 2(a+b+c)-3-j\}, \min\{2a-2j+6, 2(a+b+c-j)+2\}, \min\{2a-j+7, 2(a+b+c)-3-j\}, \min\{2a-2j+8, 2(a+b+c-j)+\right.$$

$$2\}, \dots, \min\left\{2a-j+3+2\left\lfloor\frac{n}{2}\right\rfloor, 2(a+b+c)-3-j\right\}, \min\left\{2a-2j+4+2\left\lfloor\frac{n}{2}\right\rfloor, 2(a+b+c-j)+2\right\}, \dots, \min\{2a-j+7,2(a+b+c)-3-j\}, \min\{2a-2j+8,2(a+b+c-j)+2\}, \min\{2a-j+5,2(a+b+c)-3-j\}, \min\{2a-2j+6,2(a+b+c-j)+2\}\Big)$$

$$r(v_{2,1,j,1}|W_2) = \Big(\min\{2a-j+5,2(a+b+c)-3-j\}, \min\{2a-2j+6,2(a+b+c-j)+2\}, j+1,0, \min\{2a-j+5,2(a+b+c)-3-j\}, \min\{2a-2j+6,2(a+b+c-j)+2\}, \min\{2a-j+7,2(a+b+c)-3-j\}, \min\{2a-2j+8,2(a+b+c-j)+2\}, \dots, \min\left\{2a-j+3+2\left\lfloor\frac{n}{2}\right\rfloor, 2(a+b+c)-3-j\right\}, \min\left\{2a-2j+4+2\left\lfloor\frac{n}{2}\right\rfloor, 2(a+b+c-j)+2\right\}, \dots, \min\{2a-j+7,2(a+b+c)-3-j\}, \min\{2a-2j+8,2(a+b+c-j)+2\}\Big)$$

$$\vdots$$

$$r(v_{n,1,j,1}|W_2) = \Big(\min\{2a-j+5,2(a+b+c)-3-j\}, \min\{2a-2j+6,2(a+b+c-j)+2\}, \min\{2a-j+7,2(a+b+c)-3-j\}, \min\{2a-2j+8,2(a+b+c-j)+2\}, \dots, \min\left\{2a-j+3+2\left\lfloor\frac{n}{2}\right\rfloor, 2(a+b+c)-3-j\right\}, \min\left\{2a-2j+4+2\left\lfloor\frac{n}{2}\right\rfloor, 2(a+b+c-j)+2\right\}, \dots, \min\{2a-j+7,2(a+b+c)-3-j\}, \min\{2a-2j+8,2(a+b+c-j)+2\}, \min\{2a-j+5,2(a+b+c)-3-j\}, \min\{2a-2j+6,2(a+b+c-j)+2\}, j+1,0\Big)$$

$$r(v_{1,1,j,2}|W_2) = \Big(j+1,2, \min\{2a-j+5,2(a+b+c)-3-j\}, \min\{2a-2j+6,2(a+b+c-j)+2\}, \min\{2a-j+7,2(a+b+c)-3-j\}, \min\{2a-2j+8,2(a+b+c-j)+2\}, \dots, \min\left\{2a-j+3+2\left\lfloor\frac{n}{2}\right\rfloor, 2(a+b+c)-3-j\right\}, \min\left\{2a-2j+4+2\left\lfloor\frac{n}{2}\right\rfloor, 2(a+b+c-j)+2\right\}, \dots, \min\{2a-j+7,2(a+b+c)-3-j\}, \min\{2a-2j+8,2(a+b+c-j)+2\}, \min\{2a-j+5,2(a+b+c)-3-j\}, \min\{2a-2j+6,2(a+b+c-j)+2\}\Big)$$

$$r(v_{2,1,j,2}|W_2) = \Big(\min\{2a-j+5,2(a+b+c)-3-j\}, \min\{2a-2j+6,2(a+b+c-j)+2\}, j+1,2, \min\{2a-j+5,2(a+b+c)-3-j\}, \min\{2a-2j+6,2(a+b+c-j)+2\}, \min\{2a-j+7,2(a+b+c)-3-j\}, \min\{2a-2j+8,2(a+b+c-j)+2\}, \dots, \min\left\{2a-j+3+2\left\lfloor\frac{n}{2}\right\rfloor, 2(a+b+c)-3-j\right\}, \min\left\{2a-2j+4+2\left\lfloor\frac{n}{2}\right\rfloor, 2(a+b+c-j)+2\right\}, \dots, \min\{2a-j+7,2(a+b+c)-3-j\}, \min\{2a-2j+8,2(a+b+c-j)+2\}\Big)$$

$$\vdots$$

$$r(v_{n,1,j,2}|W_2) = \Big(\min\{2a-j+5,2(a+b+c)-3-j\}, \min\{2a-2j+6,2(a+b+c-j)+2\}, \min\{2a-j+7,2(a+b+c)-3-j\}, \min\{2a-2j+8,2(a+b+c-j)+$$

$2\}, \dots, \min\left\{2a - j + 3 + 2\left\lfloor\frac{n}{2}\right\rfloor, 2(a+b+c) - 3 - j\right\}, \min\left\{2a - 2j + 4 + 2\left\lfloor\frac{n}{2}\right\rfloor, 2(a+b+c-j) + 2\right\}, \dots, \min\{2a - j + 7, 2(a+b+c) - 3 - j\}, \min\{2a - 2j + 8, 2(a+b+c-j) + 2\}, \min\{2a - j + 5, 2(a+b+c) - 3 - j\}, \min\{2a - 2j + 6, 2(a+b+c-j) + 2\}, j + 1, 2\big)$

Since $r(v_{r,1,j,k}, W_2) \neq r(v_{s,1,j,k}, W_2)$ and $W_2 \subseteq W$, then $r(v_{r,1,j,k}, W) \neq r(v_{s,1,j,k}, W)$, for all $r, s \in [1, n]$.

5. Representation $v_{i,3,j,0}$, $\forall i \in [1, n]$ for $j \in [1, c]$ with respect to $W_1 \subseteq W$ are

$$r(v_{1,3,j,0}, W_1) = \Big(a + b + j, \min\{a + b + j + 2, a + b + 2c - j + 2\}, \min\{a + b + j + 4, a + b + 2c - j + 2\}, \dots, \min\left\{a + b + j + 2\left\lfloor\frac{n}{2}\right\rfloor, a + b + 2c - j + 2\right\}, \dots, \min\{a + b + j + 4, a + b + 2c - j + 2\}, \min\{a + b + j + 2, a + b + 2c - j + 2\}\Big)$$

$$r(v_{2,3,j,0}, W_1) = \Big(\min\{a + b + j + 2, a + b + 2c - j + 2\}, a + b + j, \min\{a + b + j + 2, a + b + 2c - j + 2\}, \min\{a + b + j + 4, a + b + 2c - j + 2\}, \dots, \min\left\{a + b + j + 2\left\lfloor\frac{n}{2}\right\rfloor, a + b + 2c - j + 2\right\}, \dots, \min\{a + b + j + 4, a + b + 2c - j + 2\}\Big)$$

$$\vdots$$

$$r(v_{n,3,j,0}, W_1) = \Big(\min\{a + b + j + 2, a + b + 2c - j + 2\}, \min\{a + b + j + 4, a + b + 2c - j + 2\}, \dots, \min\left\{a + b + j + 2\left\lfloor\frac{n}{2}\right\rfloor, a + b + 2c - j + 2\right\}, \dots, \min\{a + b + j + 4, a + b + 2c - j + 2\}, \min\{a + b + j + 2, a + b + 2c - j + 2\}, a + b + j\Big)$$

Since $r(v_{r,3,j,0}, W_1) \neq r(v_{s,3,j,0}, W_1)$ and $W_1 \subseteq W$, then $r(v_{r,3,j,0}, W) \neq r(v_{s,3,j,0}, W)$, for all $r, s \in [1, n]$.

6. Representation $v_{i,3,j,k}$, $\forall i \in [1, n]$, $j \in [1, c]$, and $k \in [1,2]$ with respect to $W_3 \subseteq W$ are

$$r(v_{1,3,j,1}|W_3) = \Big(a + b + j + 1, 0, \min\{a + b + j + 3, a + b + 2c - j + 3\}, \min\{2j + 4, 2(c - j + 1)\}, \min\{a + b + j + 5, a + b + 2c - j + 3\}, \min\{2j + 6, 2(c - j + 1)\}, \dots, \min\left\{a + b + j + 1 + 2\left\lfloor\frac{n}{2}\right\rfloor, a + b + 2c - j + 3\right\}, \min\left\{2j + 4 + 2\left\lfloor\frac{n}{2}\right\rfloor, 2(c - j + 1)\right\}, \dots, \min\{a + b + j + 5, a + b + 2c - j + 3\}, \min\{2j + 6, 2(c - j + 1)\}, \min\{a + b + j + 3, a + b + 2c - j + 3\}, \min\{2j + 4, 2(c - j + 1)\}\Big)$$

$$r(v_{2,3,j,1}|W_3) = (\min\{a + b + j + 3, a + b + 2c - j + 3\}, \min\{2j + 4, 2(c - j + 1)\}, a + b + j + 1, 0, \min\{a + b + j + 3, a + b + 2c - j + 3\}, \min\{2j + 4, 2(c - j + 1)\}, \min\{a + b + j + 5, a + b + 2c - j + 3\}, \min\{2j + 6, 2(c - j + 1)\}, \dots, \min\left\{a + b + j + 1 + 2\left\lfloor\frac{n}{2}\right\rfloor, a + b + 2c - j + 3\right\}, \min\left\{2j + 4 + 2\left\lfloor\frac{n}{2}\right\rfloor, 2(c - j + 1), \dots, \min\{a + b + j + 5, a + b + 2c - j + 3\}, \min\{2j + 6, 2(c - j + 1)\}\right\}\Big))$$

$$\vdots$$

$r(v_{n,3,j,1}|W_3) = (\min\{a+b+j+3, a+b+2c-j+3\}, \min\{2j+4, 2(c-j+1)\}, \min\{a+b+j+5, a+b+2c-j+3\}, \min\{2j+6, 2(c-j+1)\}, \dots, \min\left\{a+b+j+1+2\left\lfloor\frac{n}{2}\right\rfloor, a+b+2c-j+3\right\}, \min\left\{2j+4+2\left\lfloor\frac{n}{2}\right\rfloor, 2(c-j+1), \dots, \min\{a+b+j+5, a+b+2c-j+3\}, \min\{2j+6, 2(c-j+1)\}, \min\{a+b+j+3, a+b+2c-j+3\}, \min\{2j+4, 2(c-j+1)\}, a+b+j+1, 0\right))$

$r(v_{1,3,j,2}|W_3) = \left(a+b+j+1, 2, \min\{a+b+j+3, a+b+2c-j+3\}, \min\{2j+4, 2(c-j+1)\}, \min\{a+b+j+5, a+b+2c-j+3\}, \min\{2j+6, 2(c-j+1)\}, \dots, \min\left\{a+b+j+1+2\left\lfloor\frac{n}{2}\right\rfloor, a+b+2c-j+3\right\}, \min\left\{2j+4+2\left\lfloor\frac{n}{2}\right\rfloor, 2(c-j+1)\right\}, \dots, \min\{a+b+j+5, a+b+2c-j+3\}, \min\{2j+6, 2(c-j+1)\}, \min\{a+b+j+3, a+b+2c-j+3\}, \min\{2j+4, 2(c-j+1)\}\right)$

$r(v\_(2,3,j,2) \mid W\_3) = (\min\{a+b+j+3, a+b+2c-j+3\}, \min\{2j+4, 2(c-j+1)\}, a+b+j+1, 2, \min\{a+b+j+3, a+b+2c-j+3\}, \min\{2j+4, 2(c-j+1)\}, \min\{a+b+j+5, a+b+2c-j+3\}, \min\{2j+6, 2(c-j+1)\}, \dots, \min\{a+b+j+1+2\lfloor n/2\rfloor, a+b+2c-j+3\}, \min\{2j+4+2\lfloor n/2\rfloor, 2(c-j+1), \dots, \min\{a+b+j+5, a+b+2c-j+3\}, \min\{2j+6, 2(c-j+1)\}))$

$$\vdots$$

$r(v_{n,3,j,2}|W_3) = \left(\min\{a+b+j+3, a+b+2c-j+3\}, \min\{2j+4, 2(c-j+1)\}, \min\{a+b+j+5, a+b+2c-j+3\}, \min\{2j+6, 2(c-j+1)\}, \dots, \min\left\{a+b+j+1+2\left\lfloor\frac{n}{2}\right\rfloor, a+b+2c-j+3\right\}, \min\left\{2j+4+2\left\lfloor\frac{n}{2}\right\rfloor, 2(c-j+1), \dots, \min\{a+b+j+5, a+b+2c-j+3\}, \min\{2j+6, 2(c-j+1)\}, \min\{a+b+j+3, a+b+2c-j+3\}, \min\{2j+4, 2(c-j+1)\}, a+b+j+1, 2\right\}\right)$

Since $r(v_{r,3,j,k}, W_3) \neq r(v_{s,3,j,k}, W_3)$ and $W_3 \subseteq W$, then $r(v_{r,3,j,k}, W) \neq r(v_{s,3,j,k}, W)$, for all $r, s \in [1, n]$.

7. Representation $v_{i,2,j}$, $\forall i \in [1,n]$ and $j \in [1,b]$ with respect to $W_1 \subseteq W$ are

$r(v_{1,2,j}, W_1) = \left(j+a, \min\{a+j+2, a+2b+2c+2-j\}, \min\{a+j+4, a+2b+2c+2-j\}, \dots, \min\left\{a+j+2\left\lfloor\frac{n}{2}\right\rfloor, a+2b+2c+2-j\right\}, \dots, \min\{a+j+4, a+2b+2c+2-j\}, \min\{a+j+2, a+2b+2c+2-j\}\right)$

$r(v_{2,2,j}, W_1) = \left(\min\{a+j+2, a+2b+2c+2-j\}, j+a, \min\{a+j+2, a+2b+2c+2-j\}, \min\{a+j+4, a+2b+2c+2-j\}, \dots, \min\left\{a+j+2\left\lfloor\frac{n}{2}\right\rfloor, a+2b+2c+2-j\right\}, \dots, \min\{a+j+4, a+2b+2c+2-j\}\right)$

$$\vdots$$

$r(v_{n,2,j}, W_1) = \Big(\min\{a+j+2, a+2b+2c+2-j\}, \min\{a+j+4, a+2b+2c+2-j\}, \dots, \min\left\{a+j+2\left\lfloor\frac{n}{2}\right\rfloor, a+2b+2c+2-j\right\}, \dots, \min\{a+j+4, a+2b+2c+2-j\}, \min\{a+j+2, a+2b+2c+2-j\}, j+a\Big)$

Since $r(v_{r,2,j}, W_1) \neq r(v_{s,2,j}, W_1)$ and $W_1 \subseteq W$, then $r(v_{r,2,j}, W) \neq r(v_{s,2,j}, W)$, for all $r, s \in [1, n]$.

8. Representation $v_{i,j}$, $\forall i \in [1, n]$ and $j \in [1, b]$ with respect to $W_1 \subseteq W$ are

$r(v_{1,j}, W_1) = (a+j+1, \min\{a+j+3, a+2b+2c+3-j\}, \min\{a+j+5, a+2b+2c+3-j\}, \dots, \min\left\{a+j+1+2\left\lfloor\frac{n}{2}\right\rfloor, a+2b+2c+3-j\right\}, \dots, \min\{a+j+5, a+2b+2c+3-j\}, \min\{a+j+3, a+2b+2c+3-j\})$

$r(v_{2,j}, W_1) = (min\{a+j+3, a+2b+2c+3-j\}, a+j+1, min\{a+j+3, a+2b+2c+3-j\}, min\{a+j+5, a+2b+2c+3-j\}, \dots, min\left\{a+j+1+2\left\lfloor\frac{n}{2}\right\rfloor, a+2b+2c+3-j\right\}, \dots, min\{a+j+5, a+2b+2c+3-j\})$

⋮

$r(v_{n,j}, W_1) = (min\{a+j+3, a+2b+2c+3-j\}, min\{a+j+5, a+2b+2c+3-j\}, \dots, min\left\{a+j+1+2\left\lfloor\frac{n}{2}\right\rfloor, a+2b+2c+3-j\right\}, \dots, min\{a+j+5, a+2b+2c+3-j\}, min\{a+j+3, a+2b+2c+3-j, a+j+1\})$

since $r(v_{r,j}, W_1) \neq r(v_{s,j}, W_1)$ and $W_1 \subseteq W$, then $r(v_{r,j}, W) \neq r(v_{s,j}, W)$, for all $r, s \in [1, n]$.

Since $W_1 \subseteq W_2 \subseteq W$ and $W_1 \subseteq W_3 \subseteq W$, then the representation of each vertex with respect to W_1, W_2, and W_3 will correspond to representation with respect to W. Since $r(v_a|W) \neq r(v_b|W)$ implies that $v_a = v_b$, for all $v_a, v_b \in V(Snow(n,a,b,c))$, then W is resolving set for $Snow$. Therefore, $dim(Snow(n,a,b,c)) \leq n(a+c+1)$.

Lower Bound :

Second, we will show that $dim(Snow(n,a,b,c)) \geq n(a+c+1)$. Choose the set edges of $A = \{v_{i,1,1,0} \in V(Snow(n,a,b,c)| 1 \leq i \leq n\}$ and $B = \{v_{i,1,j,0}, v_{i,3,k,0} \in V(Snow(n,a,b,c)| 1 \leq i \leq 0; 2 \leq j \leq a; 1 \leq k \leq c\}$. The sets A and B are, respectively, the set of edges connected by 3 and 2 edges of degree one, where $|A| = n$ and $|B| = n(a+c)$. By Lemma 3.2, $dim(Snow(n,a,b,c)) \geq \sum_{i=1}^{|A|}(3-1) + \sum_{i=1}^{|B|}(2-1) = n(2) + n(a-1+c) = n(a+c+1)$, so $dim(Snow(n,a,b,c)) \geq n(a+c+1)$.

Putting the bounds together gives us that $dim(Snow(n,a,b,c)) = n(a+c+1)$.

4. Conclusion

In this paper, we have defined a snowflake graph in Definition 3.3 and generalized snowflake graph in Definition 3.5. Supported with a lemma about the lower bound of the metric dimension of a such graph, we find $\dim(\text{Snow}) = 24$ proved in Theorem 3.4 and $\dim(\text{Snow}(n,a,b,c)) = n(a+c+1)$ proved in Theorem 3.6. However, it is interesting to find other measures of these graphs such as edge metric dimension, total metric dimension, partition dimension, and chromatic location number. In addition, it is worth considering that there are many other graphs that could be formed based on various snowflakes shapes other than the shape we define in this paper.

References

[1] Yu, W., Liu, Z., & Bao, X. (2020). New LP relaxations for minimum cycle/path/tree cover problems. *Theoretical Computer Science*, *803*, 71-81.

[2] Slater, P. J. (1975). Leaves of trees. *Congr. Numer*, 14(549–559), 37.

[3] Zubrilina, N. (2018). On the edge dimension of a graph. *Discrete Mathematics*, *341*(7), 2083-2088

[4] Melter, F. H., & Harary, F. (1976). On the metric dimension of a graph. *Ars Combin*, 2, 191–195.

[5] Chartrand, G., Eroh, L., Johnson, M. A., & Oellermann, O. R. (2000). Resolvability in graphs and the metric dimension of a graph. *Discrete Appl Math (1979)*, 105(1–3), 99–113.

[6] Bača, M., Baskoro, E. T., Salman, A. N. M., Saputro, S. W., & Suprijanto, D. (2011). The metric dimension of regular bipartite graphs. *Bulletin mathématique de la Société des Sciences Mathématiques de Roumanie*, 15–28.

[7] Bača, M., Baskoro, E. T., Salman, A. N. M., Saputro, S. W., & Suprijanto, D. (2011). The metric dimension of regular bipartite graphs. *Bulletin mathématique de la Société des Sciences Mathématiques de Roumanie*, 15–28.

[8] Singh, P., Sharma, S., Sharma, S. K., Bhat, V. K., Singh, P., Sharma, S., Sharma, S. K., & Bhat, V. K. (2021). Metric dimension and edge metric dimension of windmill graphs. *AIMS Mathematics 2021 9:9138*, 6(9), 9138–9153.

[9] Wu, J., Wang, L., & Yang, W. (2022). Learning to compute the metric dimension of graphs. *Appl Math Comput*, 432, 127350.

[10] Akhter, S., & Farooq, R. (2019). Metric dimension of fullerene graphs. *Electronic Journal of Graph Theory and Applications (EJGTA)*, 7(1), 91–103.

[11] Bensmail, J., Mc Inerney, F., & Nisse, N. (2020). Metric dimension: From graphs to oriented graphs. *Discrete Appl Math (1979)*.

[12] Sooryanarayana, B., Kunikullaya, S., & Swamy, N. N. (2019). Metric dimension of generalized wheels. *Arab Journal of Mathematical Sciences*, 25(2), 131–144.

[13] Klavžar, S., Rahbarnia, F., & Tavakoli, M. (2021). Some binary products and integer linear programming for k-metric dimension of graphs. *Appl Math Comput*, 409, 126420.

[14] Jiang, Z., & Polyanskii, N. (2019). On the metric dimension of Cartesian powers of a graph. *J Comb Theory Ser A*, 165, 1–14.

[15] Shao, Z., Sheikholeslami, S. M., Wu, P., & Liu, J. B. (2018). The Metric Dimension of Some Generalized Petersen Graphs. *Discrete Dyn Nat Soc*, 2018.

[16] Kelenc, A., Tratnik, N., & Yero, I. G. (2018). Uniquely identifying the edges of a graph: The edge metric dimension. *Discrete Appl Math (1979)*, 251, 204–220.

[17] Okamoto, F., Phinezy, B., & Zhang, P. (2010). The local metric dimension of a graph. *Mathematica Bohemica*, 135(3), 239–255.

[18] Knor, M., Škrekovski, R., & Yero, I. G. (2022). A note on the metric and edge metric dimensions of 2-connected graphs. *Discrete Appl Math (1979)*, 319, 454–460

[19] Sharma, K., Bhat, V. K., & Sharma, S. K. (2022). Edge Metric Dimension and Edge Basis of One-Heptagonal Carbon Nanocone Networks. *IEEE Access*, 10, 29558–29566.

[20] Klavžar, S., & Tavakoli, M. (2020). Edge metric dimensions via hierarchical product and integer linear programming. *Optimization Letters,* 15(6), 1993–2003.

[21] Sedlar, J., & Škrekovski, R. (2022). Vertex and edge metric dimensions of cacti. *Discrete Appl Math (1979)*, 320, 126–139.

[22] Sedlar, J., & Škrekovski, R. (2022). Vertex and edge metric dimensions of unicyclic graphs. *Discrete Appl Math (1979)*, 314, 81–92.

[23] Filipović, V., Kartelj, A., & Kratica, J. (2019). Edge Metric Dimension of Some Generalized Petersen Graphs. *Results in Mathematics,* 74(4), 1–15.

[24] Peterin, I., & Yero, I. G. (2019). Edge Metric Dimension of Some Graph Operations. *Bulletin of the Malaysian Mathematical Sciences Society*, 43(3), 2465–2477.
[25] Saputro, S. W., Simanjuntak, R., Uttunggadewa, S., Assiyatun, H., Baskoro, E. T., Salman, A. N. M., & Bača, M. (2013). The metric dimension of the lexicographic product of graphs. *Discrete Math*, 313(9), 1045–1051.
[26] Klavžar, S., & Tavakoli, M. (2020). Local metric dimension of graphs: Generalized hierarchical products and some applications. *Appl Math Comput*, 364, 124676.
[27] Abrishami, G., Henning, M. A., & Tavakoli, M. (2022). Local metric dimension for graphs with small clique numbers. *Discrete Math*, 345(4), 112763.
[28] Raza, H., & Ji, Y. (2020). Computing the Mixed Metric Dimension of a Generalized Petersen Graph P(n, 2). *Front Phys*, 8, 211.
[29] Raza, H., Liu, J. B., & Qu, S. (2020). On Mixed Metric Dimension of Rotationally Symmetric Graphs. *IEEE Access*, 8, 11560–11569.
[30] M Imran Bhat, S. P. (2019). On strong metric dimension of zero-divisor graphs of rings. *Korean J. Math*, 27, 563–580.
[31] Ma, X., Feng, M., & Wang, K. (2018). The strong metric dimension of the power graph of a finite group. *Discrete Appl. Math.*, 239, 159–164.
[32] Nikandish, R., Nikmehr, M. J., & Bakhtyiari, M. (2021). Metric and Strong Metric Dimension in Cozero-Divisor Graphs. *Mediterranean Journal of Mathematics*, 18(3), 1–12.
[33] Sedlar, J., & Škrekovski, R. (2022). Metric dimensions vs cyclomatic number of graphs with minimum degree at least two. *Appl Math Comput*, 427, 127147.
[34] Yi, E. (2022). On the edge dimension and the fractional edge dimension of graphs. *Discrete Appl Math (1979)*.
[35] Sharma, S. K., Raza, H., & Bhat, V. K. (2021). Computing Edge Metric Dimension of One-Pentagonal Carbon Nanocone. *Front Phys*, 9.
[36] Susilowati, L., Sa'adah, I., Fauziyyah, R. Z., Erfanian, A., & Slamin. (2020). The dominant metric dimension of graphs. *Heliyon*, 6(3), e03633.
[37] Susilowati, L., Slamin, R. A., & Rosfiana, A. (2019). The complement metric dimension of graphs and its operations. *Int. J. Civ. Eng. Technol*, 10(3), 2386–2396.
[38] Wei, M., Yue, J., & Chen, L. (2022). The effect of vertex and edge deletion on the edge metric dimension of graphs. *J Comb Optim*, 44(1), 331–342
[39] Geneson, J., Kaustav, S., & Labelle, A. (2022). Extremal results for graphs of bounded metric dimension. *Discrete Appl Math (1979)*, 309, 123–129.
[40] Mashkaria, S., Ódor, G., & Thiran, P. (2022). On the robustness of the metric dimension of grid graphs to adding a single edge. *Discrete Appl Math (1979)*, 316, 1–27.
[41] Lawson, J. E. (2009). Hands-on Science: Light, Physical Science (matter)-Chapter 5: The Colors of Light. Portage & Main Press. *Retrieved*, 6–28.
[42] Cabarkapa, A., & Djokic, L. (2019). Importance of the color of light for the illumination of urban squares. *Color Research & Application*, *44*(3), 446-453.
[43] Chartrand, G., Lesniak, L., & Zhang, P. (2015). Graphs & Digraphs. *Chapman and Hall/CRC*.
[44] Grier, Z., Soddu, M. F., Kenyatta, N., Odame, S. A., Sanders, J., Wright, L., & Anselmi, F. (2018). A low-cost do-it-yourself microscope kit for hands-on science education. In *Optics Education and Outreach V* (Vol. 10741, pp. 133-148). SPIE.