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Article

Comparison of Portfolio Mean-Variance Method with the Mean-Variance-Skewness-Kurtosis Method in Indonesia Stocks

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Abstract. In this paper, we compare the optimal portfolio weight of the mean-variance (MV) method with the mean-variance-skewness-kurtosis (MVSK) method. MV is a method to get weight on a portfolio. This method can be developed into the method of MVSK with attention to the higher-order moment of return distribution; skewness and kurtosis. In determining the weight of a portfolio, it is also important to consider the skewness and kurtosis of return distribution. This method of considering the aspects of skewness and kurtosis is called the MVSK method, with the aim of maximizing the level of return and skewness and minimizing the risks of exceeding kurtosis. The result indicate that the optimal portfolio return of all methods is the MVSK method with the lowest variance priority.

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1. Introduction

A portfolio is a combination of several assets or instruments formed by investors with the aim of obtaining profits (returns) in the future. The portfolio chosen by investors depends on their preference for return and the risk they desire[1]. To build an optimal portfolio, investors can choose one from the many options available in a set of efficient portfolios[2] [3]. A portfolio is said to be efficient if it is compared to other portfolios that meet the conditions of providing a higher expected return with the same risk or providing a smaller risk with the same expected return. In selecting a portfolio, investors generally expect high returns with low risk[4] [5]. Therefore, it is important for investors to determine a portfolio that can provide an optimum combination of return and risk.

Portfolio selection from several assets has become important problem for investors. In 1952, Harry Markowitz introduced a portfolio selection method by considering the mean as the level of expected return and variance as the level of risk in building a portfolio[6] [7]. In other words, this method considers the first two moments (mean and variance) of return distributions to find the weight of the portfolio[8] [9]. This method is known as the mean-variance (MV) Method. The assumption of the MV method is that stock returns are normally distributed[10].

However, several studies have found that stock returns are not normally distributed[10] [11] [12] where stock returns can be skewed either positive or negative with excess kurtosis. Stocks with negative skewness mean that the probability of a negative return is higher than a negative return and vice versa[13]. The third and fourth order moments, skewness and kurtosis, are considered in selecting the optimal portfolio [13] [14] [15] [16] [17]. [10] states the importance of including skewness and kurtosis in optimal portfolio selection. In[18] [19] it is explained that kurtosis is a concern and becomes very important. The method that considers aspects of skewness and kurtosis is called the mean-variance-skewness-kurtosis (MVSK) method, with the aim of maximizing the level of profit and skewness and minimizing the risk and excess kurtosis.

One of the solutions to find the value of optimization is Newton-Raphson[20][21][22]. Newton Raphson method is used to determine weight of the portfolio MVSK. Case studies will be conducted on four stocks of Bank BTN (BBTN.JK), Bank Mandiri (BMRI.JK), Indofood Sukses Makmur (INDF.JK), and Telkom (TLKM.JK) using the MV method and MVSK method. The data period of the stocks started 26 June 2016 until 02 June 2017. A comparative analysis will be carried out to obtain the optimal portfolio weights. Comparative analysis was carried out by comparing the results between the MV method and the MVSK method.

2. Experimental Section

2.1. Portfolio Optimization with Mean Variance Method

A portfolio of mean variance is defined as a portfolio that has the minimum variance among all possible portfolios that can be formed, at the same mean expected return level. On the other hand, the mean variance method uses the first-second moments of return distribution. In the mean variance portfolio, investors only invest in risk assets[23] [24]. Investors do not include risk-free assets in their portfolios. If a portfolio consists of p risky assets, a column vector $w = \left(w_1, w_2, \dots, w_p\right)^T$ which is the weight vector, which w_i denotes the weight allocated for investment in the asset to i. For p assets in a portfolio, we defined portfolio return as $R_p = w_1 r_1 + \cdots + w_p r_p$ with r_i representing return from asset to *i*.

The mean return portfolio is calculated using this formula:

$$
\begin{aligned}
E(R_p) &= E(w_1 r_1 + \cdots + w_p r_p) \\
&= w_1 E(r_1) + \cdots + w_p E(r_p)
\end{aligned}
$$

$$= \boldsymbol{w}^T \boldsymbol{\mu} \quad (1)$$

with $\boldsymbol{\mu} = (\mu_1, \mu_2, \dots, \mu_p)^T$ is a column vector in which each element represents the expected return of each asset. Then we get variance from portfolio, that is

$$\sigma_p^2 = Var\,(w_1 r_1 + \cdots + w_p r_p)$$
$$= [w_1, w_2, \dots, w_p] \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{p1} \\ \vdots & \cdots & \vdots \\ \sigma_{p1} & \cdots & \sigma_{pp} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_p \end{bmatrix}$$
$$= \boldsymbol{w}^T \boldsymbol{\Sigma}\, \boldsymbol{w}. \quad (2)$$

The mean variance method aims to optimize the weights w by minimizing the variance (risk) $\frac{1}{2}\boldsymbol{w}^{\boldsymbol{T}} \boldsymbol{\Sigma}\, \boldsymbol{w}$. Half of the quantity is only a technical reason for solving optimization problems. The optimum portfolio weighting formula can be solved by defining a portfolio that makes the risk minimal by limiting the amount of weight of the portfolio. The constraint on the mean variance method gets from the sum of vector weight elements is 1. Portfolio weight can be written in the following matrix form $\boldsymbol{w}^T \mathbf{1}_p = 1$ with $\mathbf{1}_p = (1, 1, \dots, 1)^T$ is column vector $px1$. The model can be presented as:

Objectives : $\min \frac{1}{2}\boldsymbol{w}^{\boldsymbol{T}} \boldsymbol{\Sigma}\, \boldsymbol{w}$

Constraint : $\boldsymbol{w}^T \mathbf{1}_p = 1.$ (3)

The Lagrange function L use to minimize the objective function and the given constraint as follow:

$$L = \frac{1}{2}\boldsymbol{w}^{\boldsymbol{T}} \boldsymbol{\Sigma}\, \boldsymbol{w} - \lambda(\boldsymbol{w}^{\boldsymbol{T}} \mathbf{1}_p - 1)$$

The Lagrange function is derived partially to $\boldsymbol{w}$ and is equal to zero, it will become:

$$\frac{d}{d\boldsymbol{w}}\left(\frac{1}{2}\boldsymbol{w}^{\boldsymbol{T}} \boldsymbol{\Sigma}\, \boldsymbol{w} - \lambda(\boldsymbol{w}^{\boldsymbol{T}} \mathbf{1}_p - 1)\right) = 0$$

$$\boldsymbol{w} = \lambda \boldsymbol{\Sigma}^{-\mathbf{1}} \mathbf{1}_{\boldsymbol{p}}. \quad (4)$$

then, substitute the equation $\boldsymbol{w}$ into the equation $\mathbf{1}_p{}^T \boldsymbol{w} = 1$.

$$\lambda \mathbf{1}_p{}^{\boldsymbol{T}} \boldsymbol{\Sigma}^{-\mathbf{1}} \mathbf{1}_p = 1$$

$$\lambda = (\mathbf{1}_p{}^{\boldsymbol{T}} \boldsymbol{\Sigma}^{-\mathbf{1}} \mathbf{1}_p)^{-1}. \quad (5)$$

Substituted λ to $\boldsymbol{w}$ to find the value of $\boldsymbol{w}$ as follow:

$$\boldsymbol{w} = \frac{\boldsymbol{\Sigma}^{-1} \mathbf{1}_{\boldsymbol{p}}}{\mathbf{1}_{\boldsymbol{p}}{}^{\boldsymbol{T}} \boldsymbol{\Sigma}^{-1} \mathbf{1}_{\boldsymbol{p}}} \quad (6)$$

The equation (6) use to find the weight of portfolio MV.

2.2. Portfolio Optimization with Mean-Variance-Skewness-Kurtosis (MVSK)

When optimizing the portfolio with this MVSK method, investors consider the mean, variance, skewness and kurtosis in their investment decisions or the higher-order moment. It is well known that the variance of the portfolio involves not only asset variance but also the covariance between asset returns. Because assets in the portfolio tend to move together, their profits can't be assumed to be independent. Similarly, skewness and kurtosis portfolios also involve co-skewness and co-kurtosis return assets, but in slightly different forms. The formula of co-skewness and co-kurtosis are defined as follows:

$$\sigma_{ijk} = E\left[(r_i - \mu_i)(r_j - \mu_j)(r_k - \mu_k)\right]$$
$$= E(r_i r_j r_k) - \mu_i\sigma_{jk} - \mu_j\sigma_{ik} - \mu_k\sigma_{ij} - \mu_i\mu_j\mu_k$$

and

$$\sigma_{ijkl} = E\left[(r_i - \mu_i)(r_j - \mu_j)(r_k - \mu_k)(r_l - \mu_l)\right] = E(r_i r_j r_k r_l) - \mu_i\sigma_{jkl} - \mu_j\sigma_{ikl} - \mu_k\sigma_{ijl} - \mu_l\sigma_{ijk} - \mu_i\mu_j\sigma_{kl} - \mu_i\mu_k\sigma_{jl} - \mu_i\mu_l\sigma_{jk} - \mu_j\mu_k\sigma_{il} - \mu_j\mu_l\sigma_{ik} - \mu_k\mu_l\sigma_{ij} - \mu_i\mu_j\mu_k\mu_l$$

with

r_i : *stock return i*

μ_i : *mean* for stock return *i*

Suppose there are p assets in the portfolio. The *co-skewness matrix* ($\boldsymbol{M}_3$) is a $p \times p^2$ matrix with the entry σ_{ijk}. While the matrix of cokurtosis ($\boldsymbol{M}_4$) is a $p \times p^3$matrix with the entry σ_{ijkl}. More clearly can be written

$$\boldsymbol{M}_3 = \begin{bmatrix} \sigma_{111} & \cdots & \sigma_{1p1} & \vdots & \cdots & \vdots & \sigma_{11p} & \cdots & \sigma_{1pp} \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{p11} & \cdots & \sigma_{pp1} & \vdots & \cdots & \vdots & \sigma_{p1p} & \cdots & \sigma_{ppp} \end{bmatrix}$$

and

$$\boldsymbol{M}_4 = \begin{bmatrix} \sigma_{1111} & \cdots & \sigma_{1p11} & \vdots & \cdots & \vdots & \sigma_{11p1} & \cdots & \sigma_{1pp1} & | \cdots & \cdots \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots & | \cdots & \cdots \\ \sigma_{p111} & \cdots & \sigma_{pp11} & \vdots & \cdots & \vdots & \sigma_{p1p1} & \cdots & \sigma_{ppp} & | \cdots & \cdots \end{bmatrix}$$
$$\begin{matrix} \cdots & \cdots | & \sigma_{111p} & \cdots & \sigma_{1p1p} & \vdots & \cdots & \vdots & \sigma_{11pp} & \cdots & \sigma_{1ppp} \\ \cdots & \cdots | & \vdots & \ddots & \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ \cdots & \cdots | & \sigma_{p11p} & \cdots & \sigma_{pp1p} & \vdots & \cdots & \vdots & \sigma_{p1pp} & \cdots & \sigma_{pppp} \end{matrix}$$

Furthermore, the skewness portfolio $\left(\boldsymbol{s}_{port}\right)$ and kurtosis portfolio $\left(\boldsymbol{k}_{port}\right)$ are defined as the third and fourth moments around the mean respectively:

$$\boldsymbol{s}_{port} = E(R_p - E(R_p))^3 = \boldsymbol{w}^T\boldsymbol{M}_3(\boldsymbol{w} \otimes \boldsymbol{w}) \quad (7)$$

and

$$\boldsymbol{k}_{port} = E(R_p - E(R_p))^4 = \boldsymbol{w}^T\boldsymbol{M}_4(\boldsymbol{w} \otimes \boldsymbol{w} \otimes \boldsymbol{w}) \quad (8)$$

In this case, $\otimes$ is Kronecker product and $\boldsymbol{w}^T = [w_1 \ \cdots \ w_p]$, where $\boldsymbol{w}_i$ entries in $\boldsymbol{w}^T$ is the weight for the stock *i* that will be found.

In the optimization of this MVSK portfolio, the main problem is determining the weight of funds to be invested in each stock, so that the portfolio obtained is a portfolio that has a high mean and positive skewness, and lower variance and minimizes the excess kurtosis with all the wealth invested in the whole and no money left. Mathematically, it can be expressed as:

Objectives:
Maximize $\boldsymbol{R}_p = \boldsymbol{r}^T\boldsymbol{w}$
Minimize $\boldsymbol{\sigma}^2_{port} = \boldsymbol{w}^T\boldsymbol{M}_2\boldsymbol{w}$
Maximize $\boldsymbol{s}_{port} = \boldsymbol{w}^T\boldsymbol{M_3}(\boldsymbol{w}\otimes\boldsymbol{w})$
Minimize $\boldsymbol{k}_{port} = \boldsymbol{w}^T\boldsymbol{M}_4(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w})$
Constraints $\mathbf{1}_p^T\boldsymbol{w} = 1$ (9)

From the equation (9) above, the linear combination can be formed by giving the four weighted coefficients a_1, a_2, a_3, and a_4. It can be expressed as follows;

Minimize $-a_1\boldsymbol{r}^T\boldsymbol{w} + a_2\boldsymbol{w}^T\boldsymbol{M}_2\boldsymbol{w} - a_3\boldsymbol{w}^T\boldsymbol{M}_3(\boldsymbol{w}\otimes\boldsymbol{w}) + a_4\boldsymbol{w}^T\boldsymbol{M}_4(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w})$

with constraints $\mathbf{1}_p^T\boldsymbol{w} = 1$. (10)

The Lagrange function L use to minimize the objective function and the given constraint as follow:

$$L = -a_1\boldsymbol{r}^T\boldsymbol{w} + a_2\boldsymbol{w}^T\boldsymbol{M}_2\boldsymbol{w} - a_3\boldsymbol{w}^T\boldsymbol{M}_3(\boldsymbol{w}\otimes\boldsymbol{w}) + a_4\boldsymbol{w}^T\boldsymbol{M}_4(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w}) + \lambda\left(\mathbf{1}_p^T\boldsymbol{w} - 1\right)$$

Let $a_1 = s$, $a_2 = t$, $a_3 = u$ and $a_4 = v$, with $s, u, v \geq 0$ and $t > 0$.

The Lagrange function is derived partially to w and is equal to zero, the equation become:

$$\frac{\partial}{\partial \boldsymbol{w}} - s\boldsymbol{r}^T\boldsymbol{w} + t\boldsymbol{w}^T\boldsymbol{M}_2\boldsymbol{w} - u\boldsymbol{w}^T\boldsymbol{M}_3(\boldsymbol{w}\otimes\boldsymbol{w}) + v\boldsymbol{w}^T\boldsymbol{M}_4(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w}) + \lambda\left(\mathbf{1}_p^T\boldsymbol{w} - 1\right) = 0$$

$$\boldsymbol{w} = \tfrac{1}{2t}\boldsymbol{M}_2^{-1}\left(s\boldsymbol{r} + 3u\boldsymbol{M}_3(\boldsymbol{w}\otimes\boldsymbol{w}) - 4v\boldsymbol{M}_4(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w}) - \lambda\mathbf{1}_p\right) \qquad (11)$$

We will find the value of λ with substituting $\boldsymbol{w}$ into $\mathbf{1}_p^T\boldsymbol{w} = 1$.

$$\mathbf{1}_p^T\left\{\frac{1}{2t}\boldsymbol{M}_2^{-1}\left(s\boldsymbol{r} + 3u\boldsymbol{M}_3(\boldsymbol{w}\otimes\boldsymbol{w}) - 4v\boldsymbol{M}_4(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w}) - \lambda\mathbf{1}_p\right)\right\} = 1$$

$$\tfrac{s}{\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\mathbf{1}_p}\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\boldsymbol{r} + \tfrac{3u}{\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\mathbf{1}_p}\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\boldsymbol{M}_3(\boldsymbol{w}\otimes\boldsymbol{w}) - \tfrac{4v}{\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\mathbf{1}_p}\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\boldsymbol{M}_4(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w}) - \tfrac{2t}{\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\mathbf{1}_p} = \lambda$$

The weight of portfolio MVSK become:

$$\boldsymbol{w}_{MVSK} = \frac{1}{2t}\boldsymbol{M}_2^{-1}\left(s\boldsymbol{r} + 3u\boldsymbol{M}_3(\boldsymbol{w}\otimes\boldsymbol{w}) - 4v\boldsymbol{M}_4(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w}) - \lambda\mathbf{1}_p\right)$$

with $\lambda = \tfrac{s}{\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\mathbf{1}_p}\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\boldsymbol{r} + \tfrac{3u}{\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\mathbf{1}_p}\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\boldsymbol{M}_3(\boldsymbol{w}\otimes\boldsymbol{w}) - \tfrac{4v}{\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\mathbf{1}_p}\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\boldsymbol{M}_4(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w}) - \tfrac{2t}{\mathbf{1}_p^T\boldsymbol{M}_2^{-1}\mathbf{1}_p}$

Furthermore, equation (11) will be constructed a new function $g(\boldsymbol{w})$. The function $g(\boldsymbol{w})$ will be written as follow:

$$g(\boldsymbol{w}) = \boldsymbol{w} - \left(\frac{1}{2t}\boldsymbol{M}_2^{-1}\left(s\boldsymbol{r} + 3u\boldsymbol{M}_3(\boldsymbol{w}\otimes\boldsymbol{w}) - 4v\boldsymbol{M}_4(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w}) - \lambda\boldsymbol{1}_p\right)\right) = \boldsymbol{0} \quad (12)$$

The weight of $\boldsymbol{w}_{MVSK}$ is the value of $\boldsymbol{w}$ that satisfies equation (12), which can be determined by the Newton Raphson iteration method. This iteration method uses an initial approximation and derivative value to obtain the next approximation. Portfolios in this paper consist of four stocks, which means there are four weights for each stock. The value of each weight that will be obtained by substituting it into equation (12) the result is zero, which can be mathematically written

$$g_1(w_1, w_2, w_3, w_4) = g_1(\boldsymbol{w}) = 0$$
$$g_2(w_1, w_2, w_3, w_4) = g_2(\boldsymbol{w}) = 0$$
$$g_3(w_1, w_2, w_3, w_4) = g_3(\boldsymbol{w}) = 0$$
$$g_4(w_1, w_2, w_3, w_4) = g_4(\boldsymbol{w}) = 0 \quad (13)$$

Equation (13) are function of four variables with the four variables w_1, w_2, w_3, w_4. The system of linear equation above can be form $g(\boldsymbol{w}) = \boldsymbol{0}$. Newton Raphson's formula for the multivariable problem above is,

$$\boldsymbol{w} \Leftarrow \boldsymbol{w} - J_g^{-1}(\boldsymbol{w})g(\boldsymbol{w}) \quad (14)$$

Where $J_g(\boldsymbol{w})$ is Jacobian function $g(\boldsymbol{w})$ with

$$J_g(\boldsymbol{w}) = \begin{bmatrix} \frac{\partial g_1}{\partial w_1} & \cdots & \frac{\partial g_1}{\partial w_4} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_4}{\partial w_1} & \cdots & \frac{\partial g_4}{\partial w_4} \end{bmatrix}.$$

The formula for deriving the iteration w is obtained based on the Taylor series,

$$g_i(\boldsymbol{w} + \delta\boldsymbol{w}) = g_i(\boldsymbol{w}) + \sum_j \frac{\partial g_i}{\partial w_j}\delta w_j + O(\delta\boldsymbol{w}^2) \quad \text{for } i = 1,2,3,4 \quad (15)$$

The power $\boldsymbol{w}^2$ or hihger will be ignored. For $g_i(\boldsymbol{w} + \delta\boldsymbol{w}) = \boldsymbol{0}$, it will become

$$\sum_j \frac{\partial g_i}{\partial w_j}\delta w_j = -g_i(\boldsymbol{w}) \qquad \text{for } i = 1,2,3,4.$$

Solution of linear equation δw_j, the result

$$\delta\boldsymbol{w} = -J_g^{-1}(\boldsymbol{w})g(\boldsymbol{w})$$

Finally the formula of Newton Raphson we can express it as follows:

$$\boldsymbol{w} \Leftarrow \boldsymbol{w} + \delta\boldsymbol{w} = \boldsymbol{w} - J_g^{-1}(\boldsymbol{w})g(\boldsymbol{w}) \quad (16)$$

Software R with rootSolve package will be used to solve the equation (16).

3. Results and Discussion

The data used in this case study is secondary data from Yahoo Finance. Daily stock data was retrieved from June 26 to June 2, 2017. To determine stock returns, the data is obtained from stock price data at the close price position. In this case study, the portfolio is built from four stocks in Indonesia, which are as follows:

1. PT. Bank Tabungan Negara Tbk (BBTN.JK)
2. PT. Bank Mandiri Tbk (BMRI.JK)
3. PT. Indofood Sukses Makmur Tbk (INDF.JK)
4. PT. Telkom Tbk (TLKM.JK)

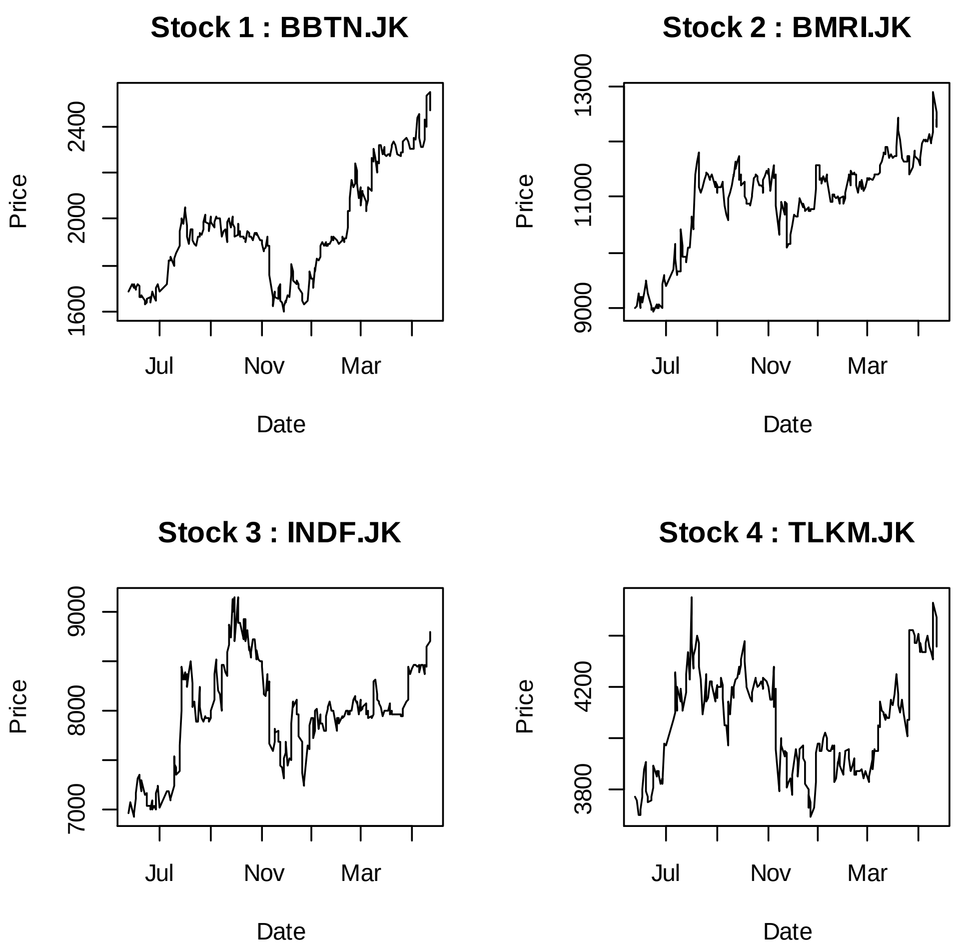

Figure 1. Historical Data Movement Close Price Each Stocks

Figure 1 shows a graph of the movement of the four stock prices used to build a portfolio. From the graph, the price movements of the four stocks fluctuated, and the graph tended to increase.

3.1 Return, Variance, Skewness and Kurtosis Each Stock

The first four orders of the moment from the data distribution are calculated, namely the return, variance, skewness and kurtosis of each stock. The calculation results can be seen in Table 1.

Table 1. Return, Variance, Skewness and Kurtosis from Each Stock

Stock	Return	Variance	Skewness	Kurtosis
BBTN.JK	0.001734	0.000371	0.397553	1.378797

BMRI.JK	0.001460	0.000304	0.269068	3.379962
INDF.JK	0.001097	0.000299	0.296570	2.494929
TLKM.JK	0.000720	0.000257	0.700152	4.132139

From Table 1, it can be seen that the four stocks have a positive average return. This shows a possibility that investors will get capital gains or profits if they form a portfolio with the four stocks. Consider the return and risk of the four stocks, it can be seen that stocks with high returns have a high level of risk, and vice versa. This indicates that the four stocks are efficient because the returns and risks are comparable. It means that these four stocks can be used to form an optimal portfolio. The data is not normally distributed because skewness is not equal to 0 and kurtosis is not equal to 3.

3.2 Normality Test of Historical Data Stock Return

In weighting the MVSK model, there is an assumption that must be fulfilled that the return data from the stock is not normally distributed. Therefore, the normality test on stock return data using the Shapiro-Wilk test with $\alpha = 0.05$ and the results obtained in Table 2 as follows.

Table 2. Normality Test of Fourth Stocks with Saphiro-Wilk Test

Stocks	P-Value	Conclusion
BBTN	2.72×10^{-5}	Return is not normally distributed
BMRI	9.88×10^{-8}	Return is not normally distributed
INDF	5.01×10^{-8}	Return is not normally distributed
TLKM	2.87×10^{-9}	Return is not normally distributed

The four stocks are not normally distributed, so the data can be used to calculate the weight of the portfolio using the MVSK method.

3.3 Comparison and Simulation Results of Optimal Weight Portfolio Mean Variance and MVSK

Stock weighting with the MV and MVSK methods uses the formula that has been obtained in equations (6) and (16). However, in the MVSK method, it is necessary to determine the values for s, t, u, v which are the weighting coefficients of the mean, variance, skewness and kurtosis, respectively. The four scenarios are

1. MVSK Priority Maximize Mean, with the coefficients $s = 4, t = 1, u = 1$ and $v = 1$.
2. MVSK Priority Minimize Variance, with the coefficients $s = 1, t = 4, u = 1$ and $v = 1$.
3. MVSK Priority Maximize Skewness, with the coefficients coefficient $s = 1, t = 1, u = 4$ and $v = 1$.
4. MVSK Priority Maximize Kurtosis, with the coefficient $s = 1, t = 1, u = 1,$ and $v = 4$.

A comparison is made between the MV method and the 4 scenarios in the MVSK method to observe the performance of the portfolio. The comparison between the weight of the portfolio using the MV and MVSK methods is as follows:

Table 3. Comparison of the Optimal Weights of MV and MVSK Portfolio

Stocks	Optimal Portfolio Weight each Method (%)				
	Mean Variance	MSVK Priority Min.	MVSK Priority Min.	MVSK Priority Max. Skewness	MVSK Priority Min. Kurtosis

		Mean	**Variance**		
BBTN.JK	0.1749	-9.7129	0.1195	-2.2750	-2.1847
BMRI.JK	0.2261	5.3930	0.3918	1.6867	1.4721
INDF.JK	0.2636	4.1236	0.3439	1.3368	1.2367
TLKM.JK	0.3352	1.1962	0.1446	0.2514	0.4758
Return	**0.1164**	-0.3581	**0.1260**	0.01655	0.0060
Risk	**0.0160**	3.1972	**0.0172**	0.2230	0.1992

In Table 3, information on the distribution of the weights of each scenario is obtained. The weights of portfolio MV and MVSK with priorities to minimize variance are positive. However, the weight of portfolio MVSK with priorities to maximize mean and kurtosis, and minimize skewness is negative. A negative weight means that investors are advised to do short selling. Short selling is recommended on BBTN.JK stocks. The higher portfolio return of scenarios is MVSK with a priority to minimize variance. The return of this priority is 0.1260% with a risk of 0.0172 %. On the other hand, the smallest portfolio risk of all methods with priority is the MV method. The risk of this scenario is 0.0160% with the return of the portfolio at about 0.1164%. The portfolio using MVSK with priority to minimize risk has the best performance compared to the other scenarios.

4. Conclusion

Based on the comparison of the MV and MVSK methods with 4 scenarios on BBTN, BMRI, INDF, and TLKM stocks, it is found that the method, with the largest return is the MVSK method with the scenario with the priority of minimizing risk having a return of 0.126%. On the other hand, the smallest portfolio risk of all methods with priority is the MV method 0.016%.

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