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Parameter Estimation STAR(1;1) Model Using Binary Weight

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Abstract. *Space Time Auto Regressive*(1;1) Model or STAR(1;1) model is a form of model that involves location and time. The STAR(1;1) model is a stationary *space time* model in mean and variance. The STAR model can be used to forecast future observations at these locations by involving the effects of observations at other nearby locations in spatial lag 1 and lag time 1 [2]. The STAR model can be written as a linear model assuming that *error* is normally distributed with zero mean and constant variance. In this research, the parameter estimation procedure for STAR model using binary weight, MKT method and STAR model for the estimation of petroleum production in 3 wells is assumed to be in a homogeneous reservoir.
Keyword: Parameter, Model or STAR(1;1), Binary Weight, stationary *space time*, Estimation

1. Introduction

Stationary time series of stochastic processes are sequences of random variables that are given time sequences. Problems related to actual time are often found in daily life, for example in the production of petroleum wells, petroleum production, cement production, rainfall and so forth.

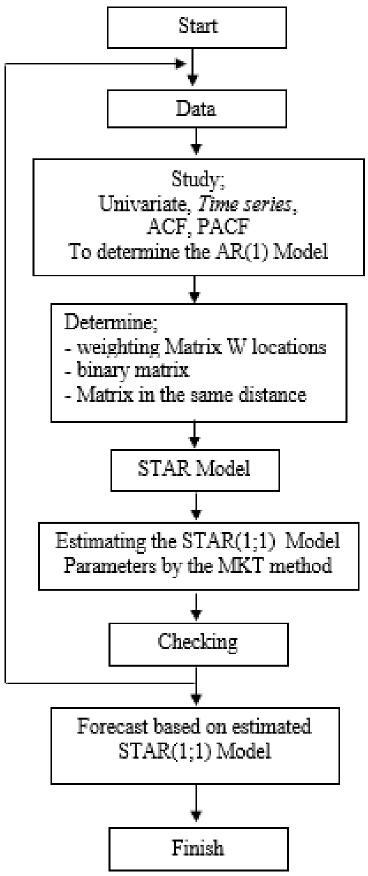
Data problems that are observed and related to time are called *time series* data. Production from PT Persero Pertamina's oil wells, Mundu, Jatibarang is a time series problem, which can be modeled with the Order 1 A*uto Regression* model or AR(1), which is a simple model, so it is better known and often used today [3]. This model can be developed if there are more than one petroleum wells through the Order 1 Auto *Regression* Vector model or VAR(1) [2]. In the VAR(1) model a correlation can be drawn between petroleum wells [4]. Further location characteristics can be illustrated through a weight matrix. The special case of the VAR(1) model is the *Space Time Auto Regression* or STAR(1;1) model [1].

2. Methods used

This study is secondary data from petroleum production, namely data of 3 petroleum production wells in PT Persero Pertamina, Mundu, Jatibarang, which was taken through collaboration between the research team of the Mathematics Department of FMIPA Unpad and the Data Section of PT Pertamina, Mundu Field, Jatibarang.

Research diagram as follows:

The flow/diagram below is the flow/diagram for research in the FMIPA Unpad Independent Research in the Mathematics Department in cooperation with PT Persero Pertamina, Mundu, Jatibarang: First start by determining the data to be processed, to process the data it is necessary to study the theory of stochastic processes; The time series, starting from the univariate; AR(1) model, ACF (Autocorrelation Function), PACF (Partial Autocorrelation Function), to establish AR(1) model. Then determine the uniform matrix, weight matrix of location W and binary matrix, parallel matrix, then; study the STAR(1;1) model, apply the STAR(1;1) model, estimate the parameters of the STAR(1;1) model with the MKT method, then check diagnosis, if there is an obstacle it will return to the next data, if it is done The diagnosis check is successful, then the process will be continued namely forecast based on the estimated STAR(1;1) model on secondary data from oil production, namely data on 3 petroleum production wells at PT Persero Pertamina, Mundu, Jatibarang, which was taken through collaboration between the Department's research team Mathematics with PT Pertamina Data Section, Mundu Field, Jatibarang.



3. Results and Discussion

Model AR(1) is the simplest univariate time series model, because it states that current time observations are influenced by observations of one time before and the *error* element [2] is written as follows:

$$z(t) = \phi(1)\ z(t\text{-}1) + e(t),\ \ e(t) \overset{iid}{\sim} N(0,\sigma^2) \qquad (1)$$

The STAR(1;1) model states that the current time observations at a particular location are influenced by an observation of a previous time at that location and the surrounding locations in a study group. For simplicity of the model, the study focused on time lag 1 and spatial lag 1 for the STAR(1;1) model in several locations [5].

The STAR(1;1) model is stated:

$$\mathbf{z}(t) = \phi_{01}\mathbf{z}(t\text{-}1) + \phi_{11}\mathbf{W}\mathbf{z}(t) + \boldsymbol{e}(t) \qquad (2)$$

and *error* vectors:

$$\boldsymbol{e}(t) \overset{iid}{\sim} \mathrm{N}(\mathbf{0},\sigma^2\mathbf{I}_N)$$

and uniform weights:

$$\mathbf{W} = \begin{pmatrix} 0 & 0,5 & 0,5 \\ 0,5 & 0 & 0,5 \\ 0,5 & 0,5 & 0 \end{pmatrix}$$

The STAR(1;1) model equation for 3 locations can be presented in the following form:

$$\begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix} = \phi_{01}\begin{bmatrix} z_1(t-1) \\ z_2(t-1) \\ z_3(t-1) \end{bmatrix} + \phi_{11}\begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix}\begin{bmatrix} z_1(t-1) \\ z_2(t-1) \\ z_3(t-1) \end{bmatrix} + \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{bmatrix} \qquad (3)$$

Equation (1) can be expressed in the form of:

$$\boldsymbol{z}(t) = [\phi_{01}\boldsymbol{I} + \phi_{11}\boldsymbol{W}]\boldsymbol{z}(t-1) + \boldsymbol{e}(t)$$

$$\mathbf{z}(t) \models \mathbf{\Phi}\mathbf{z}(t\text{-}1) + \boldsymbol{e}(t) \qquad (4)$$

with

$$\mathbf{\Phi} = [\phi_{01}\mathbf{I} + \phi_{11}\mathbf{W}]$$

Estimating the STAR(1;1) Model Parameters

The parameter estimation of the STAR(1;1) model can be done using the MKT method, because the STAR(1;1) model can be expressed as a linear model:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{e}(t),\ \ \boldsymbol{e}(t) \overset{iid}{\sim} \mathrm{N}(\mathbf{0},\sigma^2\ ) \qquad (5)$$

Estimating the linear model parameters by the MKT method for equation (5) yields:

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$$\hat{\beta} = (X' \quad X)^{-1}X'y. \qquad (6)$$

In this study, it will be shown that the parameter estimation of the STAR(1;1) model by the MKT method will give results in accordance with the equation (6).

Data of 3 petroleum production wells of PT Persero Pertamina, Mundu, Jatibarang, taken through collaboration between the Mathematics Department research team and PT Pertamina Data Section, Mundu Field, Jatibarang and stated with well 1 (V1), well 2 (V2) and well 3 (V3), from each of the data wells taken 80 data are presented in the following table;

Data Table 3 Wells, namely well 1 and well 3 of petroleum production.

	V1	V2	V3		V1	V2	V3		V1	V2	V3		V1	V2	V3
1	1162	1597	2590	**21**	2038	2230	1152	**41**	1108	1819	837	**61**	1350	1054	770
2	1358	759	155	**22**	2998	1310	1014	**42**	553	1874	818	**62**	898	1004	2887
3	1406	672	2092	**23**	1866	1864	1022	**43**	884	1532	589	**63**	943	1637	809
4	923	881	1956	**24**	1695	2283	975	**44**	1151	1401	708	**64**	874	1114	698
5	429	1028	1649	**25**	1533	2197	1224	**45**	446	2232	593	**65**	774	1155	750
6	1064	1862	2007	**26**	1035	1659	1007	**46**	268	1155	1320	**66**	817	896	1075
7	1349	1301	2003	**27**	1068	1737	1046	**47**	249	843	522	**67**	880	1252	709
8	1448	1658	2536	**28**	1086	1227	1088	**48**	666	986	397	**68**	1119	1065	949
9	2008	1607	1527	**29**	1020	1615	946	**49**	833	1227	324	**69**	759	1423	1082
10	1511	1618	1480	**30**	1192	1679	957	**50**	915	1079	921	**70**	858	1284	913
11	2261	1320	1793	**31**	1203	966	979	**51**	685	792	610	**71**	1223	1322	961
12	1446	874	1625	**32**	881	1938	957	**52**	617	944	565	**72**	895	1285	872
13	1517	1017	1548	**33**	1435	1304	877	**53**	457	589	713	**73**	581	1372	809
14	1361	690	1569	**34**	1363	1121	549	**54**	789	753	797	**74**	565	1463	895
15	1460	1053	2415	**35**	1645	1272	1057	**55**	1075	854	792	**75**	568	1811	955
16	1550	1373	2260	**36**	1450	793	693	**56**	665	951	765	**76**	525	1320	828
17	1237	2203	2577	**37**	1769	1054	1059	**57**	672	997	701	**77**	574	1105	618
18	1353	2386	1630	**38**	1290	1909	894	**58**	654	912	729	**78**	584	939	698
19	1346	2767	1600	**39**	1085	1450	1011	**59**	826	1450	776	**79**	676	966	770
20	1905	3107	1502	**40**	1490	1541	884	**60**	633	1274	570	**80**	657	1054	726

> summary

V1	V2	V3
Min.: 249.0	Min.: 589	Min.: 324.0
1st Qu.: 573.8	1st Qu.: 966	1st Qu.: 693.8
Median: 880.5	Median:1142	Median: 854.5
Mean: 977.6	Mean:1290	Mean:1031.0
3rd Qu.:1349.0	3rd Qu.:1534	3rd Qu.:1084.0
Max.:2998.0	Max.:3107	Max.:2887.0

> cor

	V1	V2	V3
V1	1.0000000	0.3912407	0.4744414
V2	0.3912407	1.0000000	0.2368895
V3	0.4744414	0.2368895	1.0000000

From the data of 3 petroleum production wells of PT Persero Pertamina, Mundu Field, Jatibarang, which were taken through collaboration between the Mathematics Department research team with the Data Section of PT Pertamina Pertamina, Mundu Field, Jatibarang and stated with wells 1 (V1), wells (V2) and wells 3 (V3), from each of the data wells taken 80 data generated graph images from each well and combined graphs from the data of the three wells and ACF and PACF graphic images of oil production data for each well, namely Well 1 (V1), Well 2 (V2) and Well 3 (V3) of petroleum production are presented as follows:

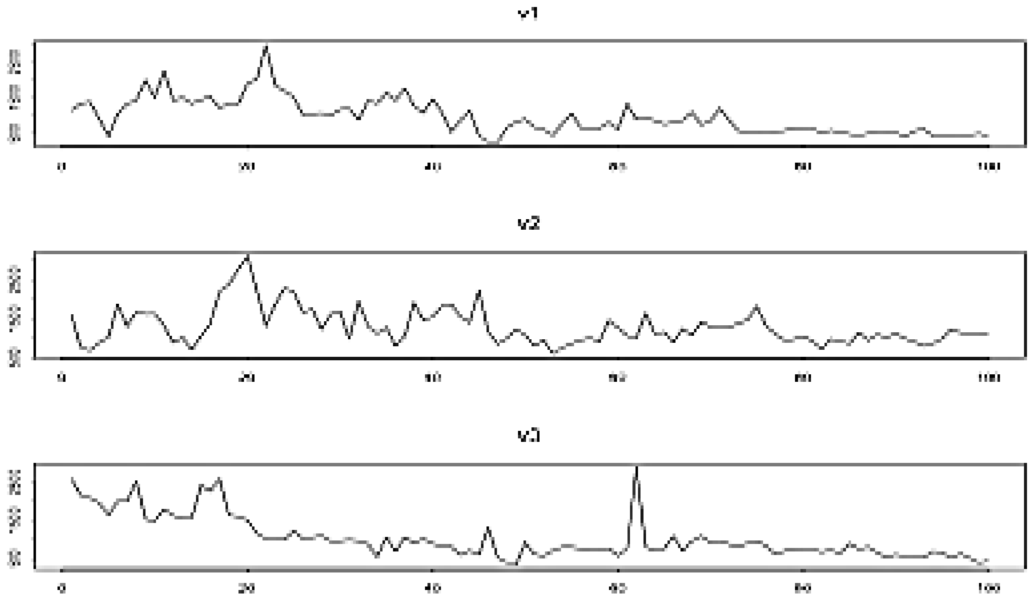

Fig 1. Oil Production Data for each Well, namely Well 1 (V1), Well 2 (V2) and Well 3 (V3)

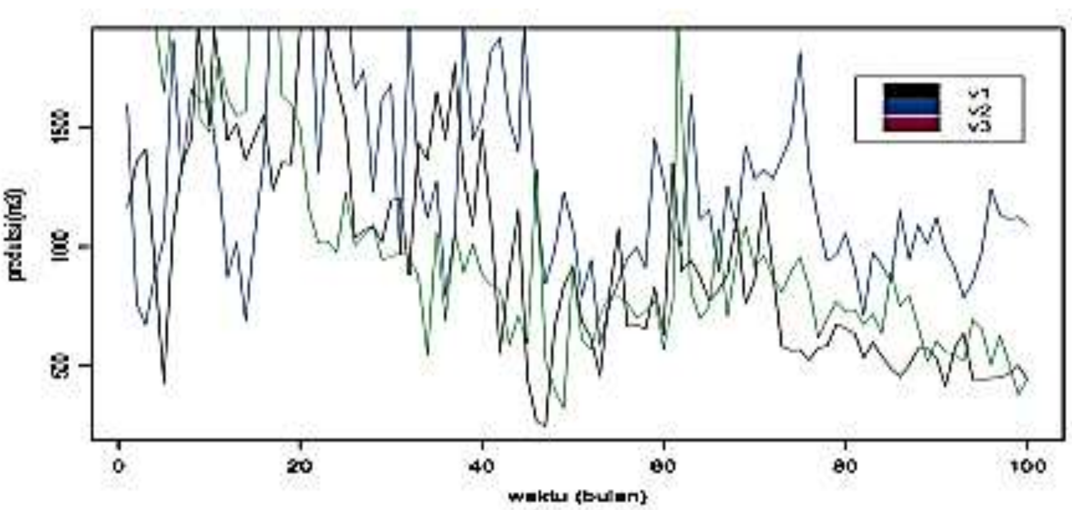

Fig 2. Combined Graph of Oil Production Data for Well 1 (V1), Well 2 (V2) and Well 3 (V3)

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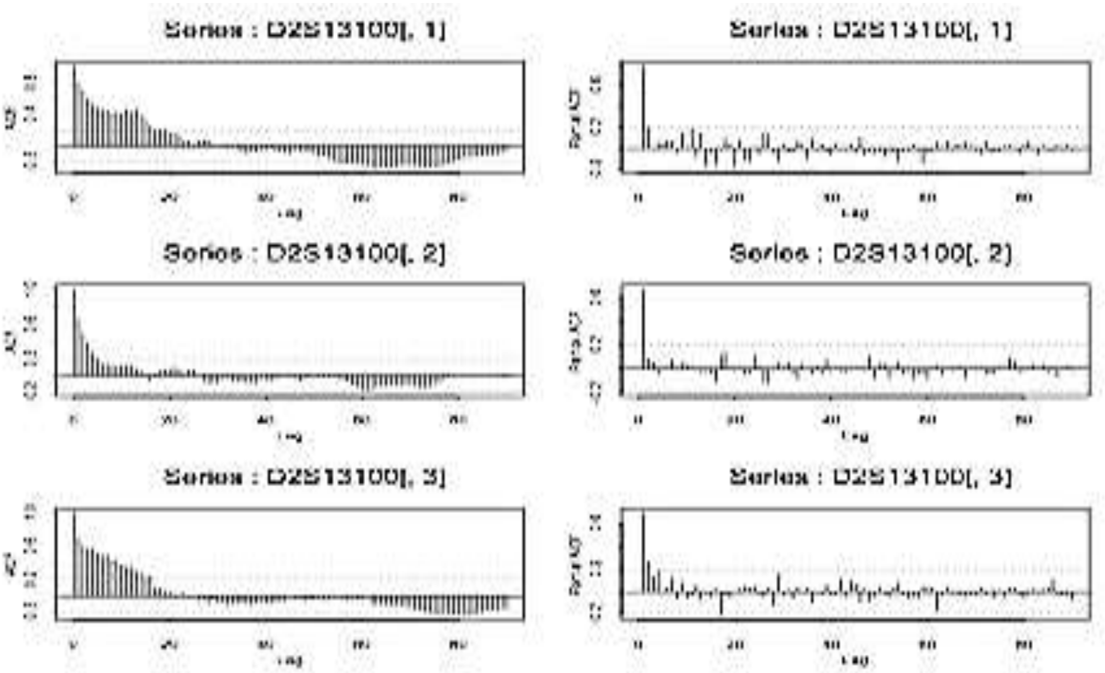

Fig 3. The ACF and PACF Graph of Oil Production Data for Well 1 (V1), Well 2 (V2) and Well 3 (V3)

MODEL STAR (TIME SERIES)

STAR Model Data

1162	1358	1406	923	429	1064	1349	1448
1597	759	672	881	1028	1862	1301	1658
2590	155	2092	1956	1649	2007	2003	2536

Weight Matrix

0	0,5	0,5
0,5	0	0,5
0,5	0,5	0

b	=	x1	x2	
1162	=	0	0	. matrix phi

X' / transpose matrix X

0	0	0	1162	1597	2590	1358	759
0	0	0	2093,5	1876	1379,5	457	756,5

X'X, product of matrices X' and X

4,63E+08	3,65E+08
3,65E+08	4,14E+08

matrix phi

0,474817
0,455369

Estimated matrix z

0	953,3151	208,1037	1181,057	1404,223	1839,287	1525,713	1112,656
0	1406,009	1574,262	1156,826	1323,096	1466,445	1117,533	966,8949
0	1857,957	555,6047	1466,445	1339,484	1114,709	1619,162	1554,422

Matrix Error (z reduced by estimated z)

1	2	3	4	5	6	7	8
1162	404,6849	1197,896	-258,057	-975,223	-775,287	-176,713	335,3444
1597	-647,009	-902,262	-275,826	-295,096	395,5549	183,4673	691,1051
2590	-1702,96	1536,395	489,5549	309,5156	892,2908	383,8379	981,5781

MSE
1,51E+08

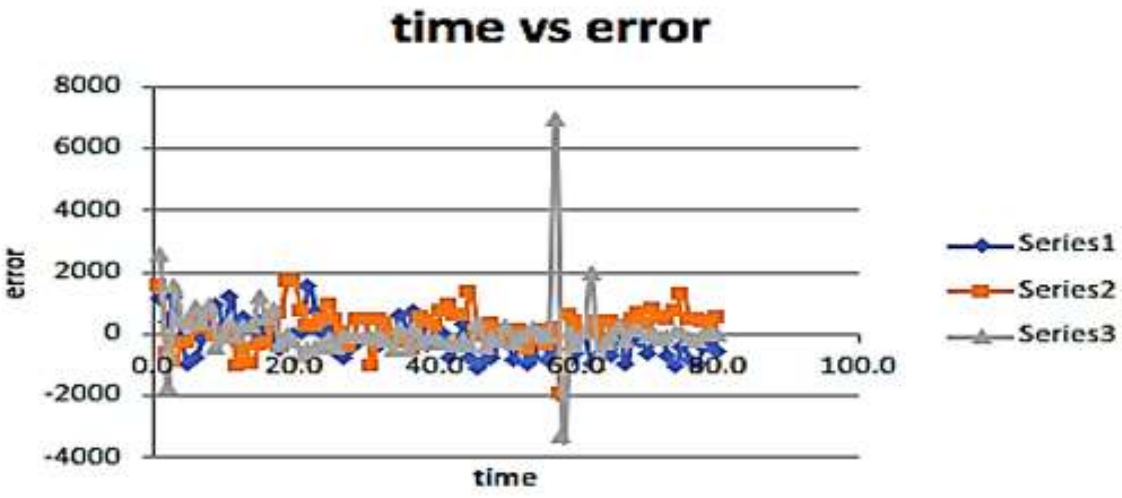

Fig 4. Star Model (Time Series) Data 3 Earth Oil Wells

Graphic Image Descriptions:

1. The blue color is a graph of Well 1 Petroleum data
2. The red color is a graph of Well 2 Petroleum data
3. The purple/grey color is a graph of Well 3 Petroleum data

4. Conclusion

The STAR(1;1) model is a multivariate time series model using a weight matrix that describes location characteristics. The STAR(1;1) model parameter $|\phi_{10}| + |\phi_{11}| < 1$. The STAR(1;1) model can be applied to various real phenomena, such as data on petroleum production. For data on STAR(1;1) oil production, it can also be used to forecast oil production at a location involving other locations around it.

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